Spring 2008 Lecture 8
SLAB ANALYSIS OF BULK FORMING PROCESSES

EXTRUSION AND DRAWING

This method entails a force balance on a slab of metal of differential thickness. This produces a differential equation where variations are considered in one direction only. Using pertinent boundary conditions, an integration of this equation then provides a solution. The assumptions involved are the following:

1. Friction does not influence the orientation of the principal axes. In the Figures below we assume that ***x, y, z*** are fixed principal axes within the deformation zone.

2. Plane sections remain plane, thus the deformation is homogeneous in regard to the determination of induced strain.

3. The principal stresses do not vary on the ***y z*** plane (see Figure below)

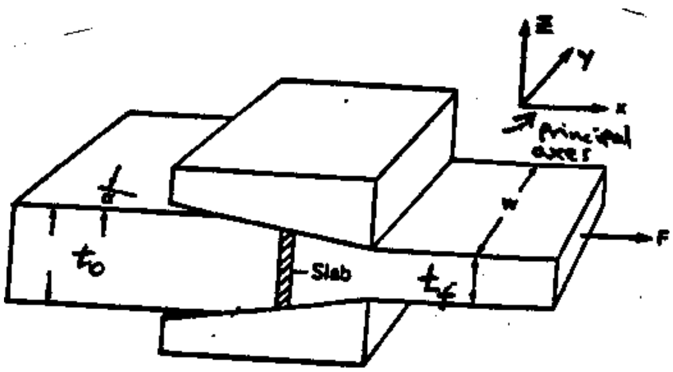

Figure 1: An example of sheet drawing showing a slab. The axes ***x, y, z*** are assumed to be principal stress axes.

Plane Strain Drawing

The drawing stress is defined as follows:

$$\frac{F}{wt_f} = \sigma_d \tag{1}$$

Using incompressibility:

$$\epsilon_x + \epsilon_y + \epsilon_z = 0 \tag{2}$$

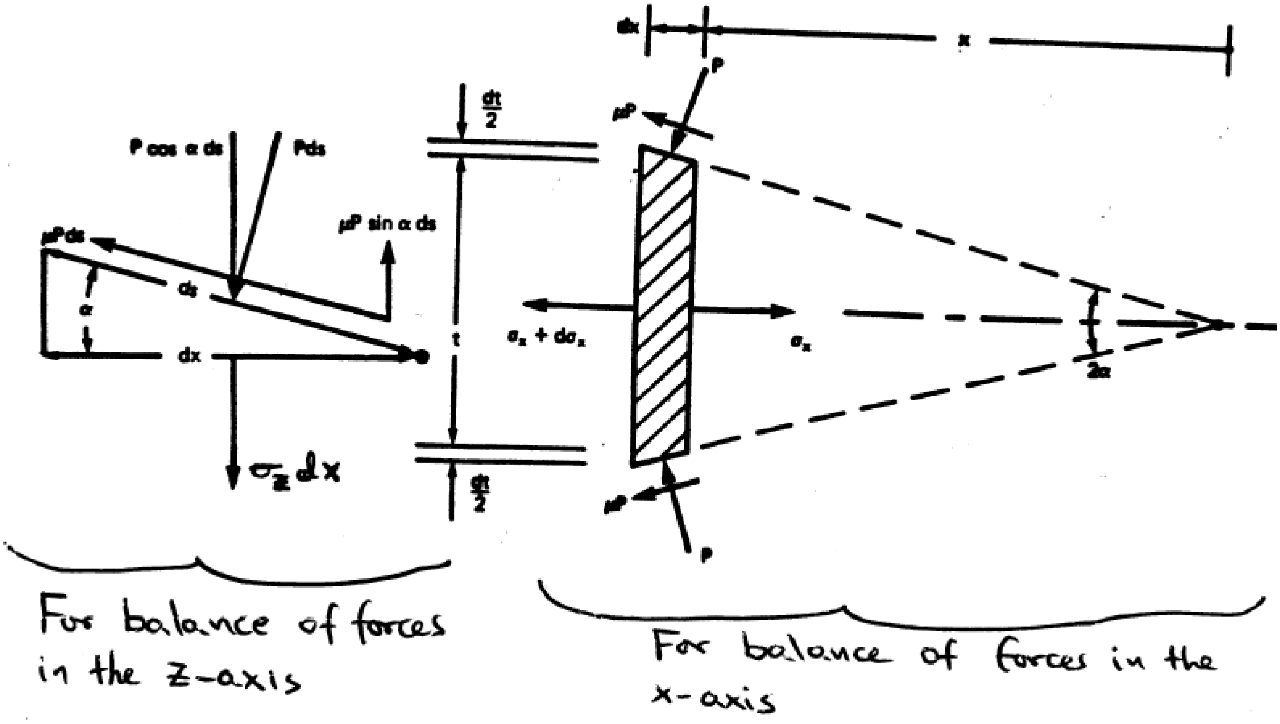

Figure 2: Sheet Drawing. Free body diagrams to calculate s_z and s_x.

and plane strain conditions,

$$\epsilon_y = 0 \tag{3}$$

results in the following:

$$\epsilon_x = -\epsilon_z \tag{4}$$

We define the homogeneous strain

$$\epsilon_h = \epsilon_x\big|_{\text{at the exit}} = -\epsilon_z\big|_{\text{at the exit}} = \ln(\frac{t_o}{t_f}) \tag{5}$$

Taking $\sum F_x = 0$ in the left free body diagram, =>

$$\sigma_z dx + (p\ ds)\cos\alpha = (\mu p\ ds)\sin\alpha \tag{6}$$

and since $\cos\alpha = dx/ds, \sigma_z = -p + \mu p\ \tan\alpha = -p\,(1 - \mu\ \tan\ \alpha)$. In general, $\mu\ \tan\alpha << 1$, so

$$\sigma_z \approx -p \tag{7}$$

indicating that s_z is compressive.
Using the flow rule for $de_y = 0$, we have that $s'_y = 0$ or finally using equation (7) and the definition of the deviatoric stress,

$$\sigma_y = \frac{\sigma_x + \sigma_z}{2} = \frac{\sigma_x - p}{2} \tag{8}$$

In terms of principal stresses, since σ_x is obviously tensile, then

$$\sigma_1 = \sigma_x,\ \sigma_2 = \frac{\sigma_x - p}{2},\ \sigma_3 = -p \tag{9}$$

Substitution in the von-Mises criterion, gives the following relation between the stress components to initiate and sustain plastic deformation (plane strain):

$$\sigma_x + p = 2\frac{Y}{\sqrt{3}} \ \text{ or } \ p = \frac{2Y}{\sqrt{3}} - \sigma_x \tag{10}$$

This equation is valid in any location inside the deformation zone - but we here assume a non-hardening material (Y = constant).

Considering equilibrium*of forces* in the *x* direction:

$$-\sigma_x wt + (\sigma_x + d\sigma_x)\,(t + dt)\,w + 2(p\,ds\,w)\text{sin}\alpha + 2(\mu\,p\,ds\,w)\text{cos}\alpha = 0 \tag{11}$$

or after neglecting higher order terms *($d\sigma_x\,dt$)*,

$$\sigma_x\,dt\ +\ t\,d\sigma_x\ +\ 2\,\mu\,p\,ds\,\text{cos}\alpha\ +\ (2\,ds\,\text{sin}\alpha)\,p = 0 \tag{12}$$

and using $dt/2 = ds$ sinα , we finally have:

$$t\,d\sigma_x\ +\ [\sigma_x +\ p\,(1\ +\ \mu\,\text{cot}\alpha)]\,dt = 0 \tag{13}$$

Let us define *B* as follows:

$$B = \mu\,\text{cot}\alpha \tag{14}$$

The equilibrium equation is now simplified as follows:

$$td\sigma_x + [\sigma_x + p\,(1 + B)]\,dt = 0 \tag{15}$$

Substituting *p* from equation (10) in the equation above gives:

$$\frac{d\sigma_x}{B\sigma_x - \frac{2Y}{\sqrt{3}}(1 + B)} = \frac{dt}{t} \tag{16}$$

The solution is based on the following assumptions.

1. An average constant value of μ , describes the full contact region.
2. The metal does not work harden, or a “mean” value of yield stress strength adequately describes any work-hardening effects; in either case,*Y* is treated as a constant.
3. The semi-die angle α is a constant

Direct integration using the conditions that σ_x =0 when $t = t_0$ and $\sigma_x = \sigma_d$ when $t = t_f$, gives

$$\sigma_d = \frac{\frac{2Y}{\sqrt{3}}(1 + B)}{B}\left[1 - \left(\frac{t_e}{t_o}\right)^B\right] \tag{17}$$

or using homogeneous strain,

$$\frac{\sigma_d}{\frac{2Y}{\sqrt{3}}} = \frac{1+B}{B}\left[1 - \exp\left(-B\epsilon_h\right)\right] \tag{18}$$

Note: Consider B —> 0 (i.e. no friction). Then using a Taylor series expansion of the exponential term in equation (18) leads to

$$\sigma_d/\frac{2Y}{\sqrt{3}} = \frac{1+B}{B}\left(1 - e^{-B\epsilon_h}\right) = \frac{1+B}{B}\left(1 - \left(1 - B\epsilon_h + \frac{B^2\epsilon_h^2}{2} + ...\right)\right) = (1+B)\left(\epsilon_h - \frac{B\epsilon_h^2}{2} + ...\right) \tag{19}$$

which as $B \to 0$ gives:

$$\frac{\sigma_d}{\frac{2Y}{\sqrt{3}}} = \epsilon_h = ln\frac{t_o}{t_f} \tag{20}$$

which is the answer also provided by the ideal work method!! From this it can be realized that *the slab analysis method simply extends the information provided by the ideal-work method to include frictional effects.*

Example.

A sheet of metal having an initial thickness of 0.100 in. and width of 12 in. is to be drawn through straight-sided dies having an included angle of 30°. If the average of the yield stress is 30 x $\sqrt{3}$ ksi and an average value for the coefficient of friction is 0.08, calculate the force needed to complete this operation for a reduction of 10%.

Solution.

$$B = 0.08 \cot 15^\circ \approx 0.3 \tag{21}$$

$$\epsilon_h = ln\left(\frac{1}{1-r}\right) = ln\left(\frac{1}{1-0.1}\right) = 0.105 \quad r = \frac{t_o - t_f}{t_o} \tag{22}$$

$$\sigma_d = \frac{2Y}{\sqrt{3}}\frac{1+B}{B}\left(1 - \exp\left(-B\epsilon_h\right)\right) \tag{23}$$

$$\sigma_d = 2\,(30)\left(\frac{1+0.3}{0.3}\right)\left[1 - \exp\left(-0.3 \times 0.105\right)\right] \tag{24}$$

$$\sigma_d = 8.08 \text{ ksi} \tag{25}$$

The drawing force, $F_d = \sigma_d\,(w)\,t_f$, so

$$F_d = 8,080\,(12)\,(0.09) = 8,730 \text{ lbf} \tag{26}$$

Wire or Rod Drawing

In terms of principal stresses,

$$\sigma_1 = \sigma_z,\ \sigma_2 = \sigma_3 = -p \tag{27}$$

where z is the main axis and the directions 2 and 3 are the hoop and radial directions. Substitution of the above equation in the von-Mises criterion, gives the following relation between the stress components to initiate and sustain plastic deformation (axisymmetric problems):

$$\sigma_z + p = Y \;\text{ or }\; p = Y - \sigma_z \tag{28}$$

For a wire or rod of circular section the basic governing equation is (why)

$$\frac{d\sigma_z}{B\sigma_z - Y(1+B)} = 2\frac{dD}{D} \quad \text{where} \quad 2\frac{dD}{D} = -d\epsilon_z \tag{29}$$

so that

$$\int_0^{\sigma_d} \frac{d\sigma_z}{B\sigma_z - Y\ (1+B)} = -\int_0^{\epsilon_h} d\epsilon_z \tag{30}$$

Integrating gives

$$\sigma_d = Y\left(\frac{1+B}{B}\right)\left[1 - \exp\left(-B\epsilon_h\right)\right] \tag{31}$$

where

$$\epsilon_h = 2\ln\frac{D_o}{D_f} \tag{32}$$

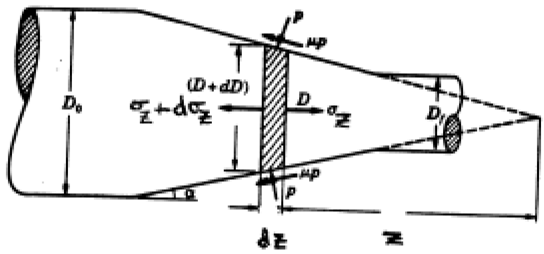

Figure 3: Slab analysis for rod drawing.

Plain strain extrusion

The equations that are different from those of plane strain drawing are listed here

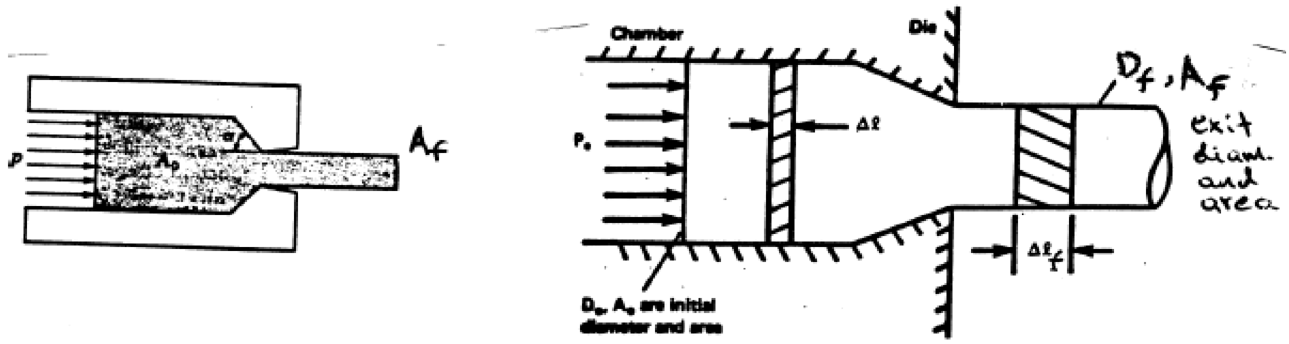

Figure 4: Plane strain and axisymmetric extrusion.

$$p_e = \frac{2Y}{\sqrt{3}}\left(\frac{1+B}{B}\right)\left[-1 + \exp\left(+B\epsilon_h\right)\right] \tag{33}$$

Axisymmetric extrusion

$$p_e = Y\left(\frac{1+B}{B}\right)\left[-1+\exp\left(+B\epsilon_h\right)\right] \tag{34}$$

Note 1: The equations above are for a non-hardening material. In the case of a hardening material, one can use the above equations (as an approximation) by taking ***Y*** to be the mean yield stress over the range of strain induced by the shape change.

Note 2: It should be noted that these analyses become unrealistic at high die angles and low reductions. Assuming that ***p*** is a principal stress is reasonable only if a is small and friction is low.

Note 3: All slab analysis calculations do not account for redundant (non-homogeneous) deformation.