Spring 2008 Lecture 9
SLAB ANALYSIS FOR FLAT ROLLING

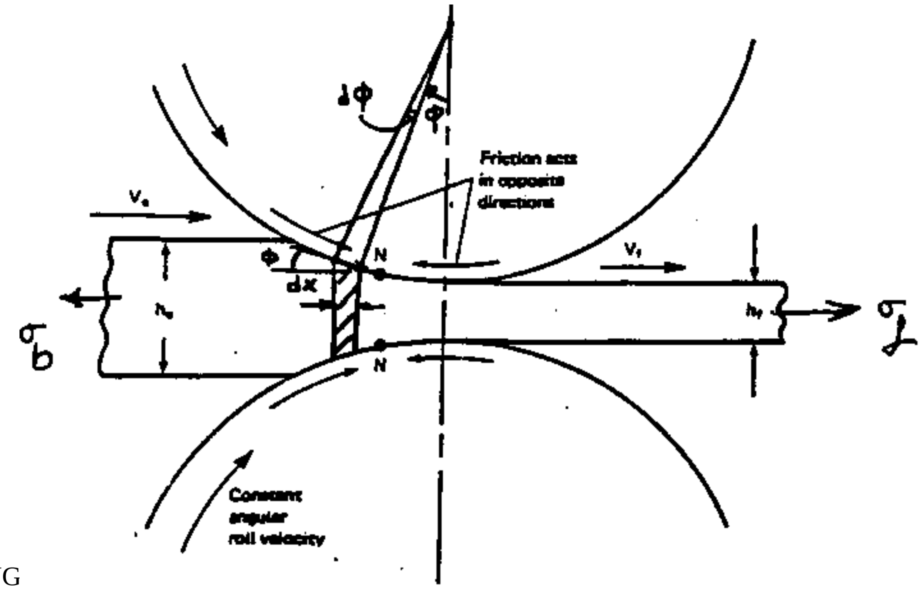

Figure 1: Schematic of flat rolling showing the neutral point

$$\begin{aligned}
\sum F_x = 0 \quad \Rightarrow \quad & (\sigma_x + d\sigma_x)(h + dh) - \sigma_x h \\
\mp \quad & (2\mu pRd\phi)\ \cos\phi \\
+ \quad & 2pRd\phi\ \sin\phi
\end{aligned} \tag{1}$$

$$\begin{aligned}
- &= \text{entry} \\
+ &= \text{exit}
\end{aligned}$$

$$\sigma_x\ h + \sigma_x dh + hd\sigma_x + \underbrace{dhd\sigma_x}_{\text{zero}} - \sigma_x h \mp 2\mu pRd\phi\ \cos\phi + 2pRd\phi\ \sin\phi = 0 \Rightarrow$$

$$\left.\begin{array}{l}
\frac{d(\sigma_x h)}{d\phi} = 2pR\left(-\sin\phi \pm \mu\ \cos\phi\right) \\
\text{Small angles:} \quad \begin{array}{l} \sin\phi \approx \phi \\ \cos\phi \approx 1 \end{array}
\end{array}\right] \Rightarrow$$

$$\frac{d\left(\sigma_x h\right)}{d\phi} = 2pR\left(-\phi \pm \mu\right) \tag{2}$$

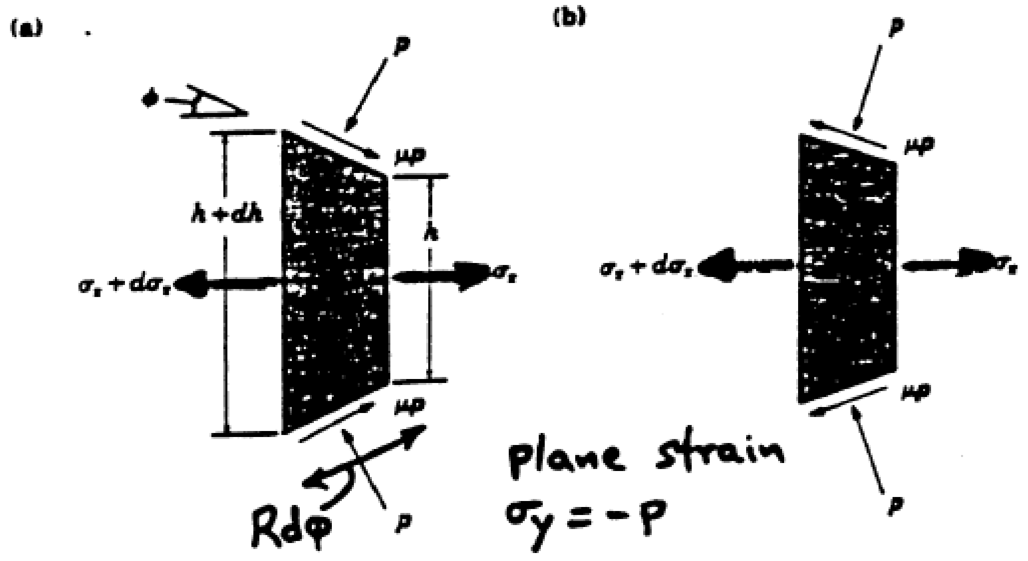

Figure 2: Stresses on an element in rolling: (a) entry zone and (b) exit zone

For small angles take $\sigma_z \simeq -p$ and for plane strain ($\epsilon_y = 0$) $\Rightarrow$

$$\underbrace{\sigma_x + p = \frac{2}{\sqrt{3}} Y}_{\text{true anywhere inside the deformation zone}} \qquad \text{(Y = Yield Stress)} \tag{3}$$

h changes with ϕ as follows:

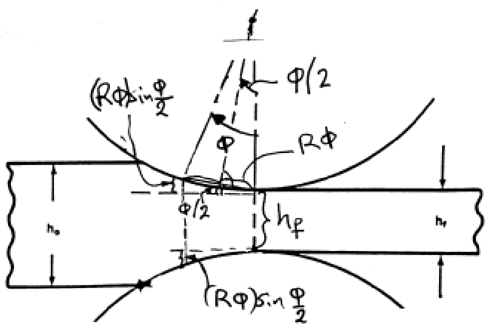

Figure 3: Approximation of h in terms of ϕ.

$$\begin{aligned} h &= h_f + 2\,(R\phi)\,\sin\frac{\phi}{2} \\ &= h_f + 2\,(R\phi)\,\frac{\phi}{2} \\ &= h_f + R\phi^2 \end{aligned} \tag{4}$$

We assume that as the material advances inside the deformation zone, its hardening behavior is such that: *Yh* = constant (so as *h* decreases, *Y* increases such that the product *Yh* remains constant!!! A ridiculous assumption that however is better than assuming that*Y* is constant inside the deformation zone!).

Returning to the equilibrium equation with the above assumption, we can write:

$$\frac{d\left(\sigma_x h\right)}{d\phi} = \frac{d\left(\frac{2}{\sqrt{3}}Y - p\right)h}{d\phi} = \underbrace{\frac{2}{\sqrt{3}}\frac{d\left(Yh\right)}{d\phi}}_{\text{zero}} - \frac{d\left(ph\right)}{d\phi}$$

$$= \quad -\frac{d\left(ph\right)}{d\phi} = -\frac{d\left(p\frac{1}{Y}Yh\right)}{d\phi} = -\frac{d\left(p/Y\right)}{d\phi}\underbrace{Yh}_{\text{const}} \tag{5}$$

Finally

$$\begin{aligned} -\frac{d\left(\frac{p}{Y}\right)}{d\phi}Yh &= 2pR\left(-\phi \pm \mu\right) \Rightarrow \\ -\frac{d\left(\frac{p}{Y}\right)}{\frac{p}{Y}} &= 2R\frac{-\phi \pm \mu}{h}d\phi \Rightarrow \\ & \boxed{-\frac{d\left(\frac{p}{Y}\right)}{\frac{p}{Y}} = 2R\frac{-\phi\pm\mu}{h_f+R\phi^2}d\phi} \end{aligned} \tag{6}$$

Let us integrate the above equation in the entry region from $j = a$ to a general angle j. Similar calculation can be applied to the exit region

$$-\int_{\text{entry}}^{\phi}\frac{d\left(\frac{p}{Y}\right)}{\frac{p}{Y}} = 2R\int_{\text{entry}}^{\phi}\frac{-\phi+\mu}{h_f+R\phi^2}d\phi \Rightarrow \tag{7}$$

$$\begin{aligned} - & \quad \left(\ln\frac{p}{Y} - \ln\frac{p}{Y}\,|_{\text{entry}}\right) = -\ln\left(h_f + R\phi^2\right) + \ln\left(h_f + R\alpha^2\right) \\ + & \quad 2R\mu\frac{1}{\sqrt{h_f R}}\left(\tan^{-1}\sqrt{\frac{R}{h}}\phi - \tan^{-1}\sqrt{\frac{R}{h}}\alpha\right) \end{aligned} \tag{8}$$

Note that in the last calculation we used the following integral formula:

$$\int\frac{dx}{a^2+b^2x^2} = \frac{1}{ab}\arctan\frac{bx}{a}, \quad (\arctan \equiv \tan^{-1}) \tag{9}$$

At the entry region using the yield condition, one can write the following:

$$\frac{p}{Y}|_{\text{entry}} = \frac{\frac{2Y}{\sqrt{3}} - \sigma_x}{Y}|_{\text{entry}} = \left(\frac{2}{\sqrt{3}} - \frac{\sigma_b}{Y_{\text{entry}}}\right) = \frac{2}{\sqrt{3}}\left(1 - \frac{\sigma_b}{Y'_{\text{entry}}}\right) \tag{10}$$

where $Y'_{\text{entry}} = \frac{2}{\sqrt{3}}Y_{\text{entry}}$.

So returning to equation (8), we can write:

$$
\begin{aligned}
& -\ln\frac{p}{Y}+\ln\frac{2}{\sqrt{3}}\left(1-\frac{\sigma_b}{Y'_{\text{entry}}}\right) \\
= \quad & -\ln\left(\overbrace{h_f+R\phi^2}^{h}\right)+\ln\left(\overbrace{h_f+R\alpha^2}^{h_o}\right) \\
+ \quad & 2R\mu\frac{1}{\sqrt{h_f R}}\left(\tan^{-1}\sqrt{\frac{R}{h}}\phi-\tan^{-1}\sqrt{\frac{R}{h}}\alpha\right)
\end{aligned}
\tag{11}
$$

Where

$$
\begin{aligned}
H &= 2\sqrt{\frac{R}{h_f}}\tan^{-1}\left(\sqrt{\frac{R}{h_f}}\phi\right) \\
H_o &= 2\sqrt{\frac{R}{h_f}}\tan^{-1}\left(\sqrt{\frac{R}{h_f}}\overbrace{\alpha}^{\text{entry}}\right)
\end{aligned}
\tag{12}
$$

Where Equation (11) is now simplified as:

$$
\begin{aligned}
-\ln \quad & \frac{\frac{p}{Y}h_o}{\frac{2}{\sqrt{3}}\left(1-\frac{\sigma_b}{Y'_{\text{entry}}}\right)h}=\mu\left(H-H_o\right)\Rightarrow \\
-\ln \quad & \frac{\frac{p}{Y'}h_o}{h\left(1-\frac{\sigma_b}{Y'_{\text{entry}}}\right)}=\mu\left(H-H_o\right) \\
\ln \quad & \frac{\frac{p}{Y'}h_o}{h\left(1-\frac{\sigma_b}{Y'_{\text{entry}}}\right)}=\mu\left(H_o-H\right)
\end{aligned}
\tag{13}
$$

Finally, the following pressure distribution is derived in the entry region:

$$
\frac{p}{Y'}=\left(1-\frac{\sigma_b}{Y'_{\text{entry}}}\right)\frac{h}{h_o}e^{\mu(H_o-H)}
\tag{14}
$$

where H and H_o are given by equation (12).

To derive the corresponding equation in the exit region, you can repeat the above calculations byintegrating equation (6) (with the bottom sign in$\pm$) from angle f to angle 0 (exit). It is also possible to derive the distribution of$p$ at the exit using equation (14) with some changes!

$$\left(\begin{array}{rcl} \text{here } h_o & \rightarrow & h_f, \quad Y'_{\text{entry}} \rightarrow Y'_{\text{exit}} \\ H_o & \rightarrow & 0 \text{ (because } \alpha = \phi_{\text{at the exit}} = 0) \end{array} \right) \tag{15}$$

$$\frac{p}{Y'} = \left(1 - \frac{\sigma_f}{Y'_{\text{exit}}}\right) \frac{h}{h_f} e^{\mu H} \tag{16}$$

Equations (14) and (16) define the complete pressure distribution in the deformation zone.

Calculation of the Neutral Point

Equate the two pressure expressions from equations (14) and (16):

$$\left(1 - \frac{\sigma_b}{Y'_{\text{entry}}}\right) \frac{h}{h_o} e^{\mu(H_o - H)} = \left(1 - \frac{\sigma_f}{Y'_{\text{exit}}}\right) \frac{h}{h_f} e^{\mu H} \tag{17}$$

$$\Rightarrow e^{\mu(2H - H_o)} = \frac{1 - \frac{\sigma_b}{Y'_{\text{entry}}}}{1 - \frac{\sigma_f}{Y'_{\text{exit}}}} \frac{h_f}{h_o} \tag{18}$$

Simplifying for the case $\sigma_b = \sigma_f = 0$ leads to:

$$H_n = \frac{1}{2}\left(H_o - \frac{1}{\mu} \ln \frac{h_o}{h_f}\right) \tag{19}$$

$$2\sqrt{\frac{R}{h_f}} \tan^{-1}\left(\sqrt{\frac{R}{h_f}} \phi_n\right) = H_n \Rightarrow \tag{20}$$

$$\boxed{\phi_n = \sqrt{\frac{h_f}{R}} \tan\left(\sqrt{\frac{h_f}{R}} \frac{H_n}{2}\right)} \tag{21}$$