# Spring 2008
# Lecture 2

# HYDROSTATIC-DEVIATORIC STRESS DECOMPOSITION AND THE CONCEPT OF STRAIN

## Hydrostatic and deviatoric stress components:

Let us consider the stress matrix representation *[a]* at a point in the body:

$$[\sigma] = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \tag{1}$$

It is convenient and useful to split the stress matrix into two parts, one called the spherical or the hydrostatic part and the other one the deviatoric part.

At first, the hydrostatic stress $a_m$ is defined as follows:

$$\sigma_m = \frac{\sigma_{xx} + \sigma_{yy} + \sigma_{zz}}{3} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \tag{2}$$

We define as hydrostatic stress state, the following:

$$[\sigma] = \begin{bmatrix} \sigma_m & 0 & 0 \\ 0 & \sigma_m & 0 \\ 0 & 0 & \sigma_m \end{bmatrix} \tag{3}$$

The name hydrostatic is used to emphasize the similar nature of the above state with the one applied on a solid cube inside a liquid (see Fig.1).

$$[\sigma] = \begin{bmatrix} \sigma_m & 0 & 0 \\ 0 & \sigma_m & 0 \\ 0 & 0 & \sigma_m \end{bmatrix} \tag{3}$$

The deviatoric stress state (matrix) *[a']* is now defined as the difference of the stress matrix*[a]* (equ.(1)) from the hystrostatic stress matrix given by equation (3) ie.

$$[\sigma'] = \begin{bmatrix} \sigma_{xx} - \sigma_m & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} - \sigma_m & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} - \sigma_m \end{bmatrix} = \begin{bmatrix} \frac{(2\sigma_{xx} - \sigma_{yy} - \sigma_{zz})}{3} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \frac{(2\sigma_{yy} - \sigma_{xx} - \sigma_{zz})}{3} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \frac{(2\sigma_{zz} - \sigma_{xx} - \sigma_{yy})}{3} \end{bmatrix} \tag{4}$$

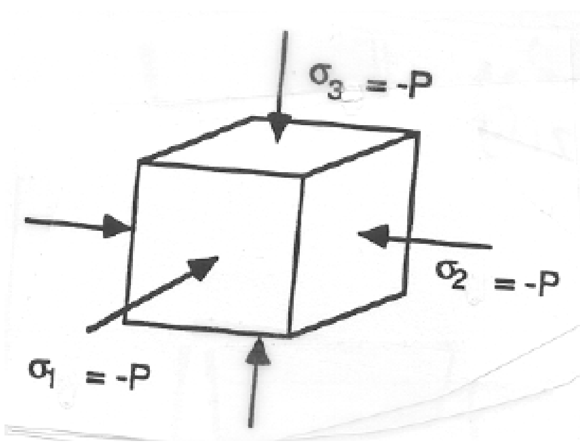


Figure 1: A hydrostatic stress state with $p = -\sigma_m$

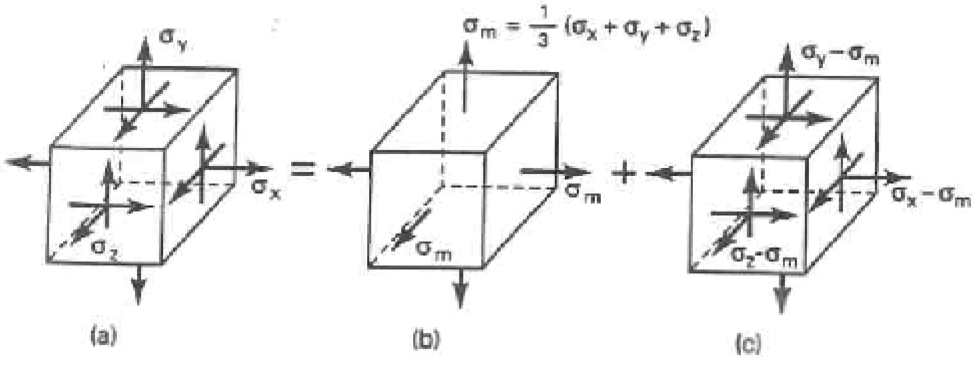


Figure 2: The decomposition of stress in hydrostatic and deviatoric parts.

In terms of the principal stresses, the principal deviatoric stress components can be written as follows:

$$[\sigma'] = \begin{bmatrix} \sigma_1 - \sigma_m & 0 & 0 \\ 0 & \sigma_2 - \sigma_m & 0 \\ 0 & 0 & \sigma_3 - \sigma_m \end{bmatrix} = \begin{bmatrix} \frac{(2\sigma_1 - \sigma_2 - \sigma_3)}{3} & 0 & 0 \\ 0 & \frac{(2\sigma_2 - \sigma_1 - \sigma_3)}{3} & 0 \\ 0 & 0 & \frac{(2\sigma_3 - \sigma_1 - \sigma_2)}{3} \end{bmatrix} \quad (5)$$

A graphical representation of the above decomposition of the stress matrix is shown in Fig.2.We will later see that the hydrostatic stress part is related to the change of volume of a material during deformation, while the deviatoric part is responsible for the induced distortion.

# Uniaxial Strain:

Consider a bar of length $L_o$.By applying forces as shown in Fig.3(a), we extend the length of the bar by an amount $\Delta L = L - L_o$.
We define the nominal or engineering strain $e$ as follows (see Fig.3).

$$e \equiv \frac{\Delta L}{L_o} \quad (6)$$

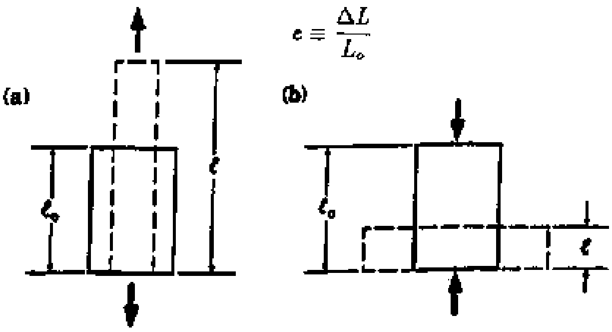


Figure 3: Definition of the uniaxial strain (a) Tensile and (b) Compressive $L_o$ is the original length and $\Delta L$ the length change after the load application.

In addition to the above normal nominal strain, one can define the engineering shear strain $\gamma$ as the change of angle as shown in Fig.4.For small angle change, we can write:

$$\gamma \equiv \frac{a}{b} \quad (7)$$

Figure 5 shows both the deformed and undeformed configurations of an infinitesimal cube under uniaxial tension and pure shear.
Both of the above definitions are applicable only for small deformations (e.g. equation (6) is applicable for stretches less than 2 % in tension).Why is that?

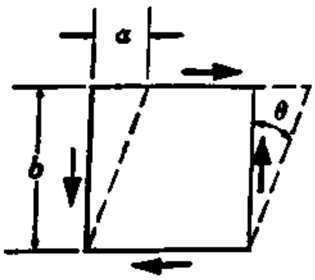


Figure 4: Shear strains are used to define change of angles upon application of forces.

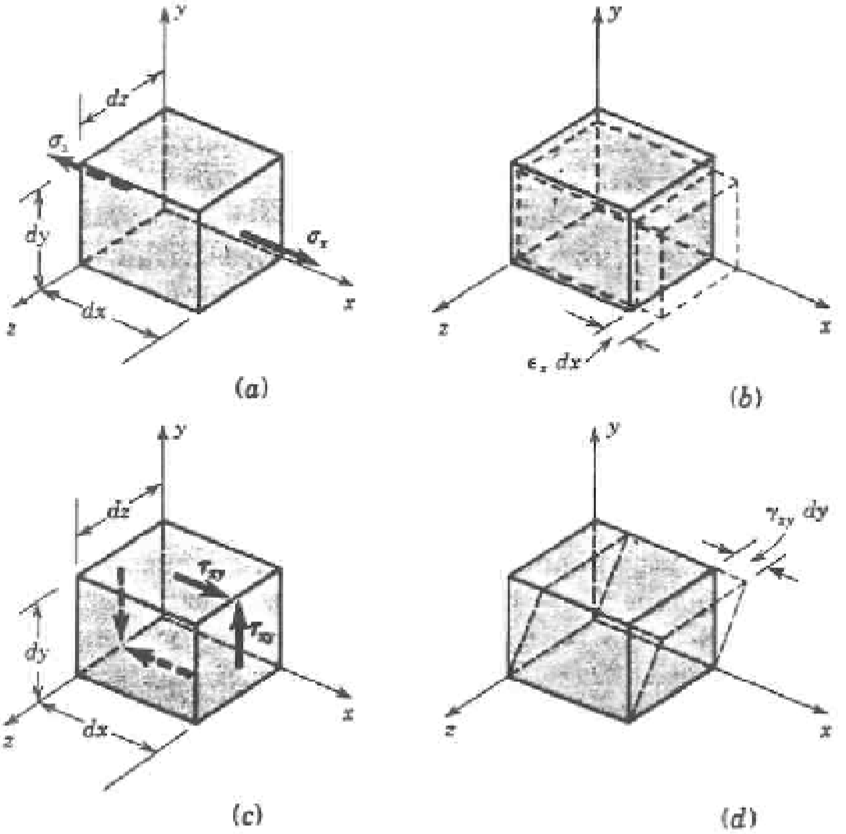


Figure 5: Infinitesimal element subjected to: (a) uniaxial tension with the resulting deformation and (b) pure shear with the resulting deformation.

Let us use two examples to demonstrate why the above definitions are not appropriate for large deformations. Let us consider a tensile experiment in which a specimen of length $L_o$ is elongated to length $2L_o$. Using equ.(6), the predicted engineering strain is e= $(2L_o - L_o)/ L_o=1$ . Let us now assume that the above extension from $L_o$ to $2L_o$ is achieved in two stages; from $L_o$ to $1.5L_o$ and from $1.5L_o$ to $2L_o$. In the first stage, the engineering strain is $e_1= (1.5\ L_o - L_o)/ L_o=0.5$ while in the second stage $e_2= (2\ L_o -1.5\ L_o)/ L_o=0.333$. One expects that, $e_1 + e_2$ should be equal to $e$ = 1. Unfortunately, $e_1 + e_2 = 0.833$ (This implies that the engineering strain is not additive).

As a second example, let us consider again the tension experiment that elongates a specimen of length $L_o$ to a length $2L_o$. We showed that $e$ = 1. Let us now imagine a uniaxial compression experiment (see Fig.3(b)) with $e$ = -1. What should the new length be? A guess would be $L = L_o/2$. However, $e = -1 = ( L - L_o)/ L_o$ from which we derive that $L$ = 0!

Obviously there is a problem while trying to use the engineering strain definitions for problems involving large deformations.

To correct the above problems, we will define the so calle true strain Consider again uniaxial extension that is performed in several small steps from the original length $L_o$ to the final desired length $L$. In each step, we define an incremental true strain $de$ as follows:

$$d\epsilon = \frac{dL}{L} \tag{8}$$

where $dL$ is the differential change in length during that step and $L$ is the length at the beginning of the step. The total strain would be the sum (integral!) of all the $de$'s from the initial length $L_o$ to the final length $L$, i.e.

$$\epsilon = \int_{L_o}^{L} d\epsilon = \int_{L_o}^{L} \frac{dL}{L} \quad \text{or} \tag{9}$$

$$\epsilon \equiv ln\left(\frac{L}{L_o}\right) \tag{10}$$

This equation defines the *true strain* e. Note that for small $\Delta L = L - L_o$ and since $ln(1 + x) \sim x$ for $x << 1$, we have:

$$\epsilon = ln\left(1 + \frac{\Delta L}{L_o}\right) \simeq \frac{\Delta L}{L_o} = e \tag{11}$$

i.e. at small strains the nominal and true strains are equal. Let us return to the two examples examined earlier:

In the first example, the sum of $e_1 + e_2 = ln\, 1.5\, L_o/L_o + ln\, 2\, L_o/1.5\, L_o = ln2\, L_o/L_o = ln2$ as expected (i.e. the true strain is additive).

In the second example, note that $e = -ln2 = ln\, L_o/L_o$, from which we conclude that $L = L_o/2$ as expected.

One can also define a true shear strain as the tangent of the deformation angle rather than the angle itself (but fortunately we will never have to work with large shear strains in this course!).

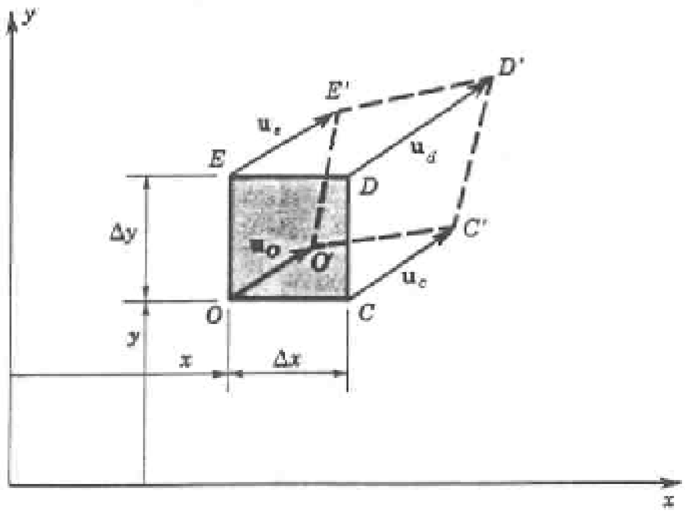


Figure 6: Plane strain deformation in the xy plane of a small element of a continuous body. The displacements of any point *(x, y)* are only (continuous) functions of the coordinates *x* and *y*.

# Two-Dimensional Strain:

Most of the following analysis is only applicable to small deformation problems.We use the notation *e* for strain to emphasize this assumption and reserve the notation e for large strains.
To simplify the presentation we only discuss the definition of the two-dimensional strain components but an extension to 3D will be apparent. Consider an infinitesimal square section $\Delta x$ x $\Delta y$ of a body in the *xy* plane. Assume that deformation occurs only on the *xy* plane and that it is only a function of the *x* and *y* coordinates. We call later call this deformation state a plane strain state in the *xy* plane. Figure 6 shows the deformed body in terms of the displacements for this case of plane strain.

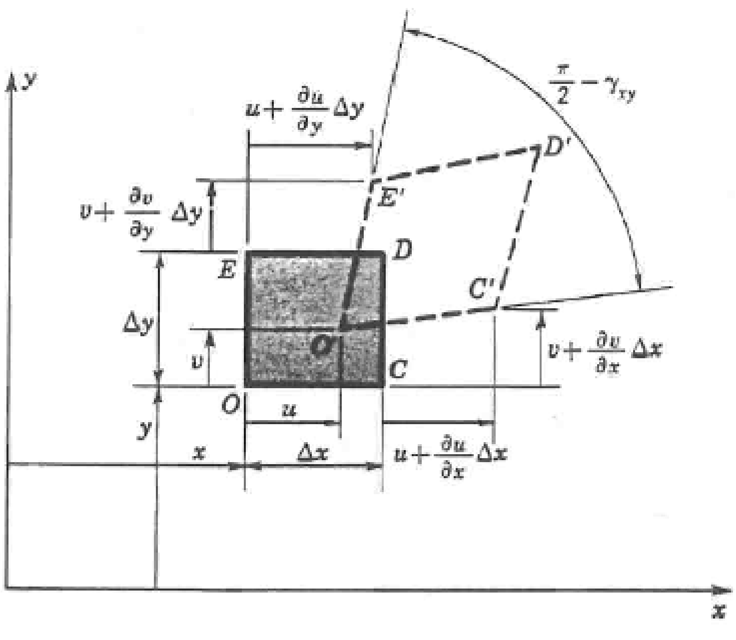

Figure 7: Plane strain deformation expressed in terms of the components $u$ and $v$ and their partial derivatives. Here $u$ and $v$ are the displacements of point $O$ (it will thus be more precise to denote them as $u_0$ and $v_0$, respectively). Similarly the derivatives $\partial u/\partial x$, $\partial v/\partial y$, etc. shown in this figure are computed at point $O$. The displacements $u(x,y)$ and $v(x,y)$ of any other point $(x,y)$ are functions of $x$ and $y$ and can be approximated using a Taylor series expansion around $O$. The sizes $\Delta x$ and $\Delta y$ of the square section are assumed small.
Here, we will define the (small) strain components $e_{xx}$ and $e_{yy}$ at point $O$ as the relative changes of the lengths $\Delta x$ and $\Delta y$ in the x and y axes, respectively:

$$
\begin{aligned}
e_{xx} &= \lim_{\Delta x \to 0} \frac{O'C' - OC}{OC} = \lim_{\Delta x \to 0} \frac{[\Delta x + (\partial u/\partial x)\,\Delta x] - \Delta x}{\Delta x} = \frac{\partial u}{\partial x} \\
e_{yy} &= \lim_{\Delta y \to 0} \frac{O'E' - OE}{OE} = \lim_{\Delta y \to 0} \frac{[\Delta y + (\partial \nu/\partial y)\,\Delta y] - \Delta y}{\Delta y} = \frac{\partial \nu}{\partial y}
\end{aligned} \tag{12}
$$

For more details consult Fig.7.

Similarly, one can define the shear strain component $\gamma_{xy}$ as follows:

$$
\begin{aligned}
\gamma_{xy} &= \lim_{\Delta x \to 0, \Delta y \to 0} \left( \frac{\pi}{2} - \lfloor C'O'E' \right) = \lim_{\Delta x \to 0, \Delta y \to 0} \left\{ \frac{\pi}{2} - \left[ \frac{\pi}{2} - \frac{(\partial \nu/\partial x)\,\Delta x}{\Delta x} - \frac{(\partial u/\partial y)\,\Delta y}{\Delta y} \right] \right\} \\
&= \frac{\partial \nu}{\partial x} + \frac{\partial u}{\partial y}
\end{aligned} \tag{13}
$$

Extending these ideas to three-dimensions and assuming a displacement field $u(x,y,z)$, $v(x,y,z)$ and $w(x,y,z)$, we define the 9 strain components as follows:

$$
\begin{aligned}
e_{xx} &= \frac{\partial u}{\partial x}, \quad \gamma_{xy} = \frac{\partial \nu}{\partial x} + \frac{\partial u}{\partial y}, \quad \gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}, \\
\gamma_{yx} &= \frac{\partial u}{\partial y} + \frac{\partial \nu}{\partial x}, \quad e_{yy} = \frac{\partial \nu}{\partial y}, \quad \gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial \nu}{\partial z}, \\
\gamma_{zx} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \quad \gamma_{zy} = \frac{\partial \nu}{\partial z} + \frac{\partial w}{\partial y}, \quad e_{zz} = \frac{\partial w}{\partial z}
\end{aligned} \tag{14}
$$

Important note: We will not use these expressions often in this course as our strains will be in the large deformation regime. For such applications we will use as strain measures an extension of our logarithmic strain introduced earlier in one-dimension. For example, the strain $e_{xx}$ will be defined as $e_{xx} = ln(L/L_0)$, where the length $L_0$ was lying in the $x$-axis before the application of the loads. Similarly, we will define $e_{yy}$ and $e_{zz}$. We will not need to work with large shear strains in this course!

## Plane Strain Problems:

Consider a long prismatic member subject to lateral loading (for example, a cylinder under pressure), held between *fixed*, smooth, rigid planes (see Fig.8).Assume the external forces to be functions of the $x$ and $y$ coordinates only. As a consequence, we expect all cross sections to experience identical deformation, including those sections near the ends. The frictionless

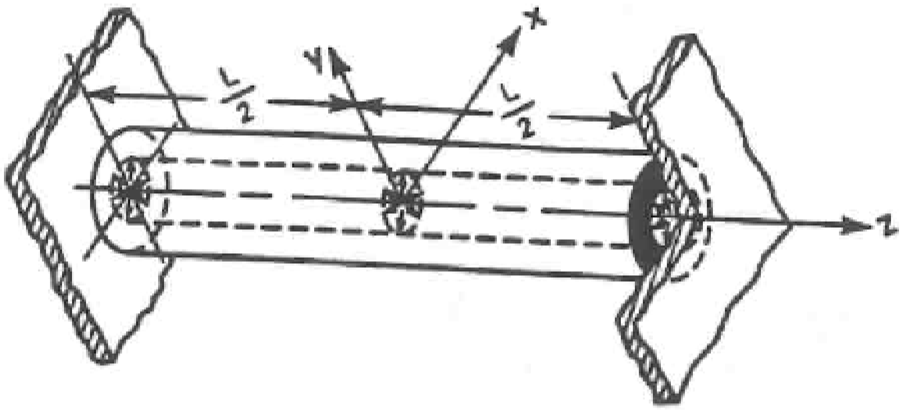


Figure 8: Plane strain in the xy plane. All strain components out of the xy plane are zero, i.e. $e_{zz}$ = $\gamma_{yz}$ = $\gamma_{yz}$=0.The same definition is applicable to large strains but you should use the appropriate (logarithmic) large strain measures.

nature of the end constraint permits *x, y* deformation, but precludes *z* displacement; that is, $w = 0$ at $z = \pm L/2$.Considerations of symmetry dictate that *w* must also be zero at midspan. Symmetry arguments can again be used to infer that $w = 0$ at $\pm L/4$, and so on, until every cross section is taken into account. For the case described, the strain depends on *x* and *y* only:

$$
\begin{aligned}
e_{xx} &= \frac{\partial u}{\partial x}, \qquad e_{yy} = \frac{\partial \nu}{\partial y}, \qquad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial \nu}{\partial x} \\
e_{zz} &= \frac{\partial w}{\partial z} = 0, \quad \gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = 0, \quad \gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial \nu}{\partial z} = 0
\end{aligned}
\tag{15}
$$

The latter expressions depend on $\partial u/\partial z$ and $\partial v/\partial z$ vanishing, since *w* and its derivatives are zero. A state of plane strain (on the xy plane) has thus been described wherein each point on the xy remains in this plane, following application of the load.

## The Strain Matrix:

Just as the state of stress is described by a nine-term array, we can define the strain matrixas:

$$
[e] = \begin{bmatrix} e_{xx} & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{yx} & e_{yy} & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{zx} & \frac{1}{2}\gamma_{zy} & e_{zz} \end{bmatrix} = \begin{bmatrix} e_{xx} & \epsilon_{xy} & e_{xz} \\ e_{yx} & \epsilon_{yy} & e_{yz} \\ e_{zx} & e_{zy} & e_{zz} \end{bmatrix} \tag{16}
$$

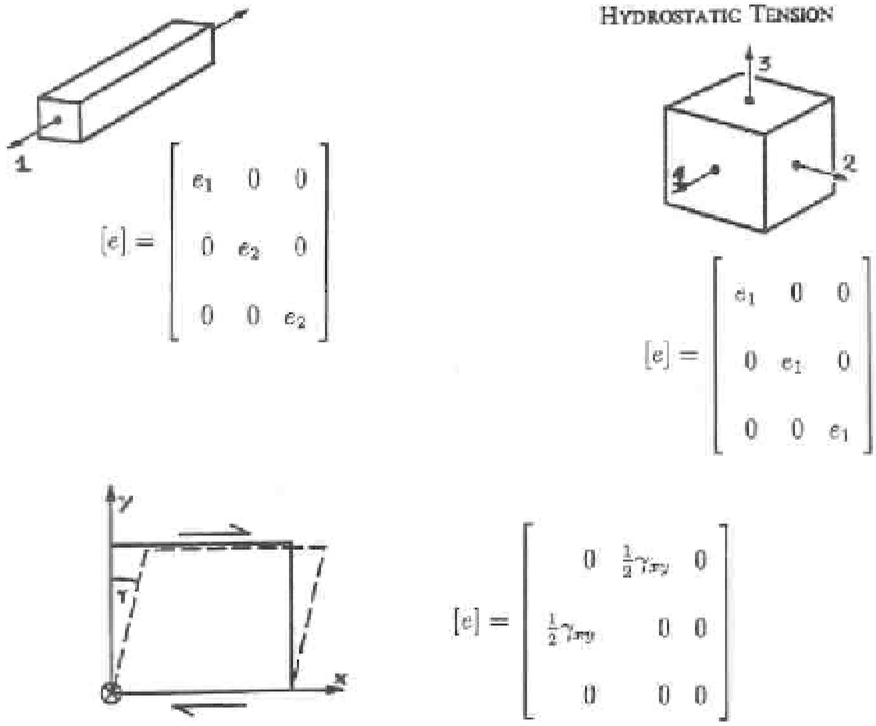


Figure 9: Examples of strain states (a) Uniaxial tension for an isotropic material (b) equal hydrostatic tension in the three Cartesian axes and (c) shear.
where:

$$e_{xy} = \frac{1}{2}\gamma_{xy}, \quad e_{yz} = \frac{1}{2}\gamma_{yz}, \quad e_{xz} = \frac{1}{2}\gamma_{xz} \tag{17}$$

These nine strain components are needed to define the deformation of a cube.The strain matrix is symmetric, e.g. $e_{xy}$ = $e_{yx}$, etc.Also, we occasionally write $e_x$ instead of $e_{xx}$, etc. Note in the definition of the strain matrix we used half of the engineering shear strains.This is to allow us to use transformation equations from one coordinate system to another as we did for stress (you do not need to worry why we introduced the strain matrix like this, but be sure that you know what strain you are using, e.g. $\gamma_{xy}$ or $e_{xy}$).

The test cube can always be rotated into one particular orientation where all the shear strain components vanish. These principal strain directions are denoted as 1, 2 and 3, while the principal strains are denoted as $e_1,\ e_2,\ e_3$.

Figure 9(a, b) shows three simple strain states in terms of principal strain components.

Later in the course, we will see that for an isotropic material (e.g. a linear isotropically elastic material), the principal strain directions are the same as the principal stress directions.

The strain matrix can be written in terms of the principal strain components as follows:

$$e = \begin{pmatrix} e_1 & 0 & 0 \\ 0 & e_2 & 0 \\ 0 & 0 & e_3 \end{pmatrix} \quad (18)$$

Note that there is an important property of the strain component transformation:

$$e_{xx} + e_{yy} + e_{zz} = e_1 + e_2 + e_3 \quad (19)$$

In the condition of plane strain examined earlier, one of the principal strain components $e_1$, $e_2$, $e_3$ is zero, for example the following strain matrix corresponds to plane strain in the plane 12:

$$e = \begin{pmatrix} e_1 & 0 & 0 \\ 0 & e_2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (20)$$

Similar equations are true for the logarithmic strain.

## Relative Change of Volume in Terms of Strain Components:

Consider a unit cube (dimensions 1 x 1 x 1 ) ) along the principal strain directions. Under loading, the cube will deform to another cube of dimensions $(1 + e_1)$ x $(1 + e_2)$ x $(1 + e_3)$.The *dilatation*, $\Delta$, is defined as the relative change of volume of the cube, i.e.

$$\Delta = \frac{\Delta V}{V} = \frac{(1+e_1)(1+e_2)(1+e_3) - 1}{1 \times 1 \times 1} \simeq e_1 + e_2 + e_3 \quad (21)$$

or

$$\Delta = \frac{\Delta V}{V} = e_1 + e_2 + e_3 = e_{xx} + e_{yy} + e_{zz} \quad (22)$$

Note that if the deformation preserves volume (incompressible deformation), then

$$e_1 + e_2 + e_3 = e_{xx} + e_{yy} + e_{zz} = 0 \quad (23)$$

In case of plane strain conditions (e.g. $e_{zz} = 0$, the above condition can be further simplified as $e_{xx} = -e_{yy}$).

A final note: Most of the deformations to be examined in this class are large incompressible deformations. For such deformations, we will approximate the incompressibility condition as follows:

$$\epsilon_1 + \epsilon_2 + \epsilon_3 = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz} = 0 \quad (24)$$

or more precisely using true strain increments *(de = dl/l)* as follows:

$$d\epsilon_1 + d\epsilon_2 + d\epsilon_3 = d\epsilon_{xx} + d\epsilon_{yy} + d\epsilon_{zz} = 0 \quad (25)$$

The proof of this equation is straightforward (take a cube with sizes $l1 \times l_2 \times l3$ lying on the principal strain axes. Assuming that the volume does not change during deformation, i.e. $d(l_1 \times l_2 \times l_3) = 0$, you can show that $dl_1/l_1 + dl_2/l_2 + dl_3/l_3 = 0$ which is precisely the equation above.

# Transformation of Strain Components in Plane Strain Conditions:

Similarly to the transformation equations derived for the stress components, we can derive transformation equations for the strain components. Note the similarity between the normal strains $e_{xx}$, $e_{yy}$ and $e_{zz}$ and the normal stresses $\sigma_{xx}$, $\sigma_{yy}$ and $\sigma_{zz}$ as well as the similarity between $e_{xy}(={}^{1/2}\gamma_{xy})$, $e_{yz}(={}^{1/2}\gamma_{yz})$ and $e_{xz}(={}^{1/2}\gamma_{xz})$ with the shear stresses $\tau_{xy}$, $\tau_{yz}$ and $\tau_{xz}$.

Using the notation of Fig.10, we define the strains $e_{x'x'}$, $\gamma_{x'y'}$, *etc.* as follows:

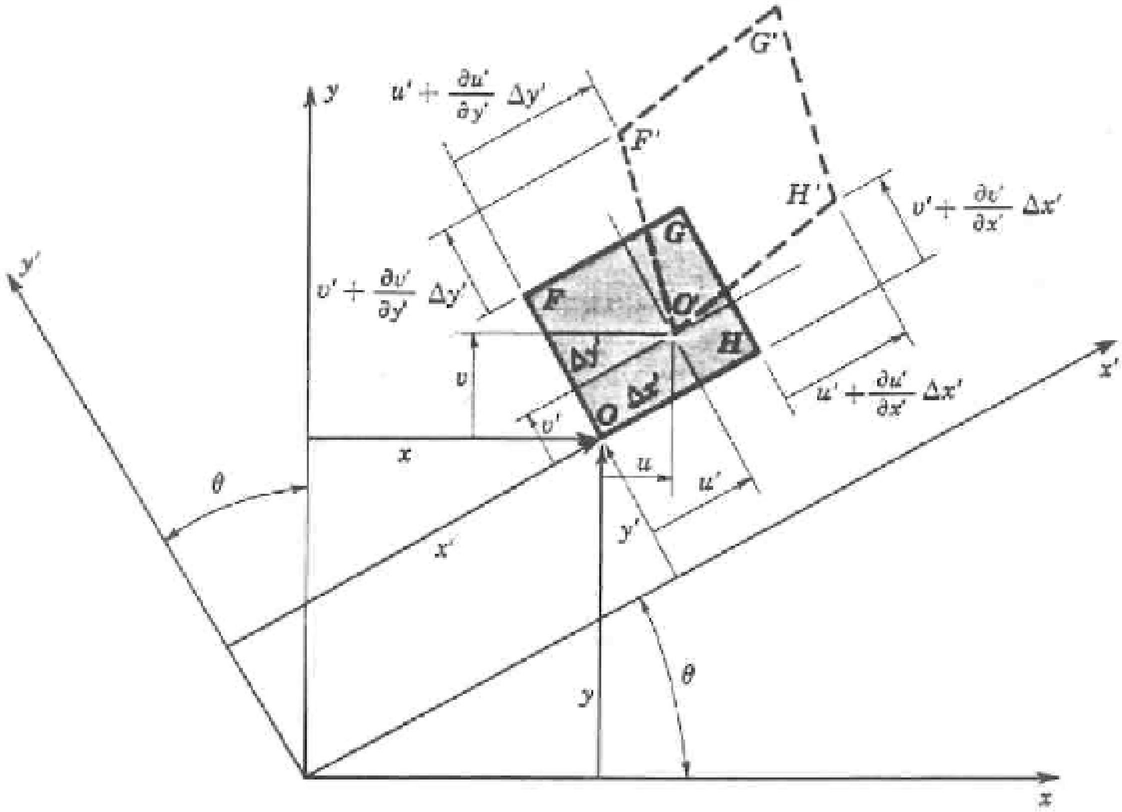


Figure 10: Deformation of a small element with sides originally parallel to $x'$ and $y'$ axes. $u'$ and $v'$ are here the displacements of point O in the directions of the axes $x'$ and $y'$, respectively.

$$e_{x'x'} = \frac{\partial u'}{\partial x'}, \; e_{y'y'} = \frac{\partial v'}{\partial y'}, \; \gamma_{x'y'} = \frac{\partial u'}{\partial y'} + \frac{\partial v'}{\partial x'} \quad (26)$$

The final strain transformation equations have the following form:

$$e_{x'x'} = \frac{e_{xx}+e_{yy}}{2} + \frac{e_{xx}-e_{yy}}{2} \quad \cos 2\theta \quad + \frac{\gamma_{xy}}{2}\sin 2\theta$$
$$e_{y'y'} = \frac{e_{xx}+e_{yy}}{2} - \frac{e_{xx}-e_{yy}}{2} \quad \cos 2\theta \quad - \frac{\gamma_{xy}}{2}\sin 2\theta$$
$$\frac{\gamma_{x'y'}}{2} = -\frac{e_{xx}-e_{yy}}{2} \quad \sin 2\theta \quad + \frac{\gamma_{xy}}{2}\cos 2\theta \tag{27}$$

The principal strain directions (where $\gamma_{x'y'} = 0$) are found from:

$$\tan 2\theta_p = \frac{\gamma_{xy}}{e_{xx}-e_{yy}} \tag{28}$$

Similarly, the magnitudes of the principal strains are

$$e_{1,2} = \frac{e_{xx}+e_{yy}}{2} \pm \sqrt{\left(\frac{e_{xx}-e_{yy}}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} \tag{29}$$

The maximum shearing strains are found on planes 45̊ relative to the principal planes and are given by

$$\gamma_{\max} = \pm 2\sqrt{\left(\frac{e_{xx}-e_{yy}}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \pm\left(e_1 - e_2\right) \tag{30}$$

Note that the above transformation equations are only valid for small strain. We will not need the transformation equations for the logarithmic strain as we will always try to work on principal strain axes!!

## Mohr’s Circle for Small Strain:

Because we have concluded that the transformation properties of stress and strain are identical, it is apparent that a *Mohr’s circle for strain* may be drawn and that the construction technique does not differ from that of Mohr’s circle for stress (see Fig.11).

- In Mohr’s circle for strain, the normal strains are plotted on the horizontal axis, positive to the right.
- When the shear strain is positive, the point representing the $x$ axis strains is plotted a distance $\gamma/2$ *below* the $e$ line, and the $y$ axis points a distance $\gamma/2$ *above* the $e$ line, and vice versa when the shear strain is negative.

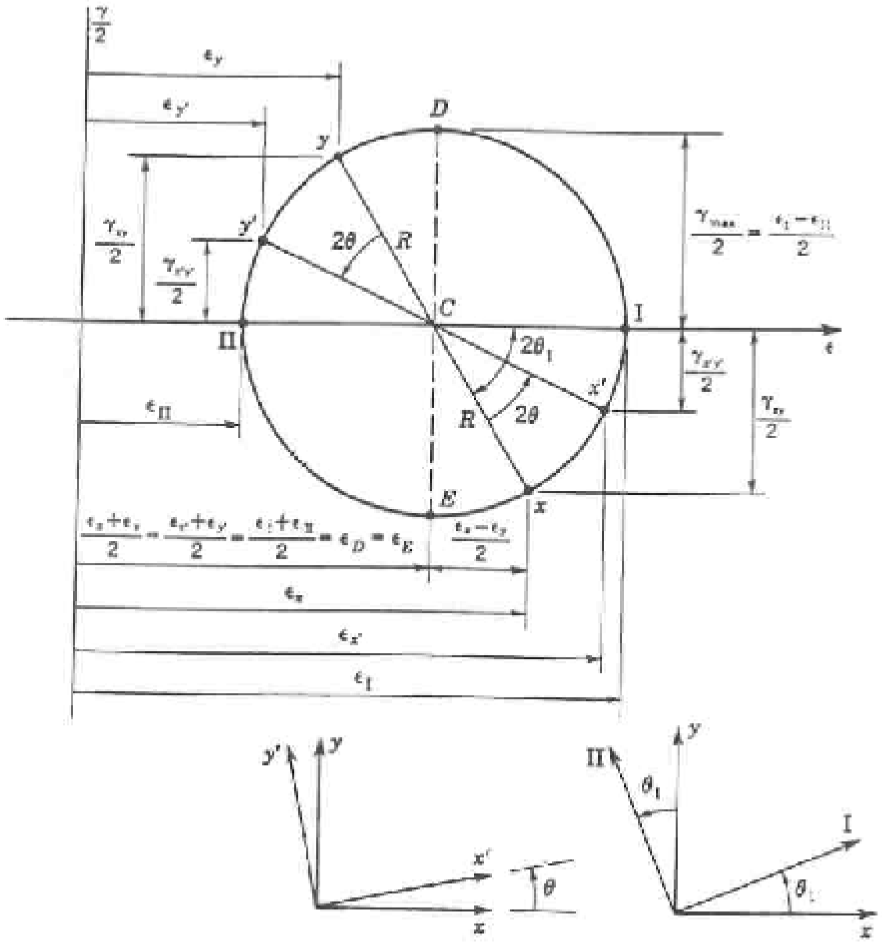

Figure 11: The Mohr circle for plane strain problems.

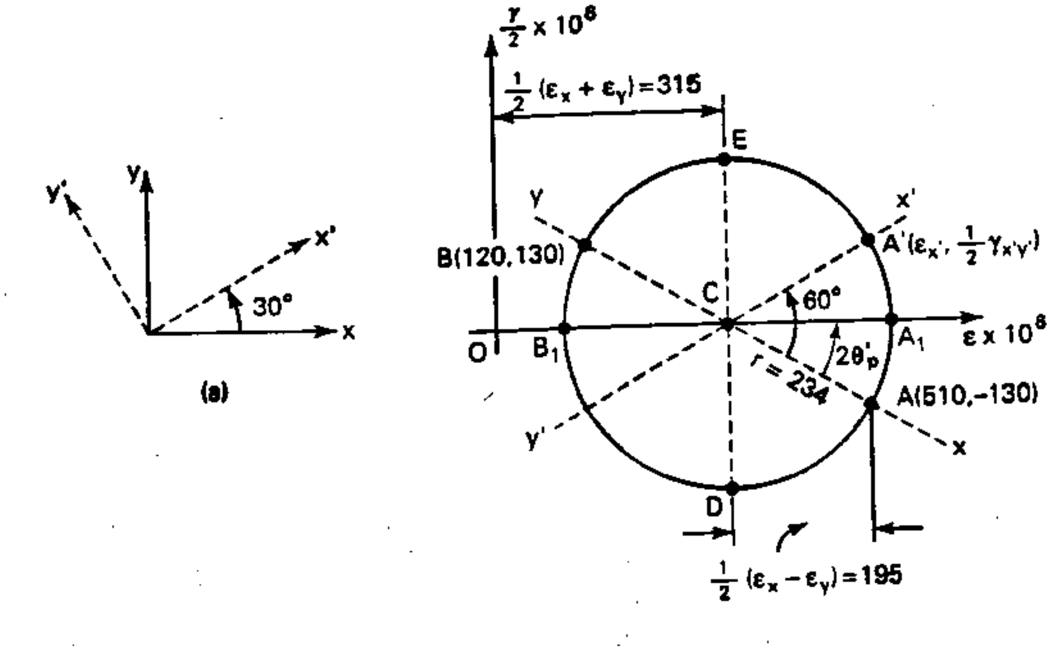


Figure 12: The Mohr circle for the example problem.

Example of a Mohr circle design for plane strain:

The state of strain at a point on a steel plate is given by $e_{xx}$ = 510$\mu$, $e_{yy}$ = *120$\mu$*, and $\gamma_{xy}$ = 260$\mu$ (here $\mu$ = $10^{-6}$). Let us determine, using Mohr's circle of strain,

- the state of strain associated with axes *x′,y′*, which make an angle $\theta$ = 30° with the axes *x,y;*
- the principal strains and directions of the principal strain axes;
- the maximum shear strains and associated normal strains.

A sketch of Mohr's circle of strain is shown in Figure 12, constructed by determining the position of point *C* at 1/2($e_{xx}$ + $e_{yy}$) and *A* at ($e_{xx}$,*1/2*$\gamma_{xy}$), from the origin *O*. Note that *1/2*$\gamma_{xy}$ is positive, so point *A*, representing the *x*-axis strains, is plotted below the *e* axis (or *B* above). Carrying out calculations similar to that for Mohr's circle of stress, the required quantities are determined. The radius of the circle is $r$ = $(195 + 130\ )^2$ $\mu$ = 234$\mu$, and the angle $2\theta_p'$ = $\tan^{-1}$ *(130/95)* = 33.7°.
At a position 60° counterclockwise from the *x* axis lies the *x′* axis on Mohr's circle, corresponding to twice the angle on the plate.The angle $A'CA_1$ is 60° - 33.7° = 26.3°.The strain components associated with *x′y′* are therefore:

$$\begin{aligned} e_{x'x'} &= 315\mu + 234\mu \cos 26.3^\circ = 525\mu \\ e_{y'y'} &= 315\mu - 234\mu \cos 26.3^\circ = 105\mu \\ \gamma_{x'y'} &= -2\,(234\ \mu\ sin 26.3^\circ) = -207\mu \end{aligned} \tag{31}$$

The shear strain is taken as negative because the point representing the *x* axis strains, *A′* is above the *e* axis. The negative sign indicates that the angle between the element faces *x′* and *y′* at the origin increases.
The principal strains, represented by points *A1* and *B*1, on the circle, are found to be

$$\begin{aligned} e_1 &= 315\mu + 234\mu = 549\mu \\ e_2 &= 315\mu - 234\mu = 81\mu \end{aligned} \tag{32}$$

The axes of $e_1$ and $e_2$ are directed at 16.85° and 106.85° from the *x* axis, respectively.
Finally, points *D* and *E* represent the maximum shear strains. Thus

$$\gamma_{\max} = \pm 468\mu \tag{33}$$

Observe from the circle that the axes of maximum shear strain make an angle of 45 with respect to the principal axes. The normal strains associated with the axes of $\gamma_{\max}$ are equal, represented by *OC* on the circle: 315$\mu$.