Spring 2008 Lecture 4
THE IDEAL WORK METHOD
FOR THE ANALYSIS OF FORMING PROCESSES

In general the prediction of external forces needed to cause metal flow is needed. Such prediction is difficult due to uncertainties introduced from frictional effects and non-homogeneous deformation as well as from not knowing the true manner of strain hardening.
Each solution method is based on several assumptions. The easiest method is the ideal work method. The work required for deforming the workpiece is equated to the external work. The process is considered ideal in the sense that the external work is completely utilized to cause deformation only. Friction and non-homogeneous deformation are neglected.

AXISYMMETRIC EXTRUSION AND DRAWING

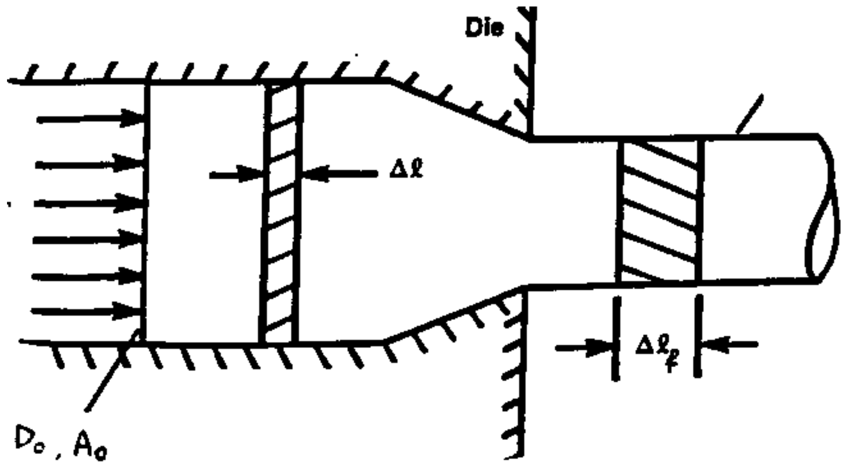

Figure 1: Illustration of direct or forward extrusion assuming ideal deformation.

Let us consider axisymmetric extrusion (Fig. 1) where the diametral area is reduced from A_0 to A_f. The ideal work is:

$$w_i = \int_0^{\bar{\epsilon}_f} \bar{\sigma} d\bar{\epsilon} \tag{1}$$

where $\bar{\epsilon}_f = \epsilon_{axial_f} = ln\frac{A_o}{A_f} = ln\frac{1}{1-r}$ and r is the percent area reduction, i.e. $r = \frac{A_o - A_f}{A_o}100\%$

Note that the final axial strain is usually called the homogeneous strain and denoted as ϵ_h , ie.

$\epsilon_{axial_f} = \epsilon_h = ln\frac{1}{1-r}$

Assuming:

$$\bar{\sigma} = K\bar{\epsilon}^n$$

We finally can write

$$w_i = \int_0^{\bar{\epsilon}_f} \bar{\sigma} d\bar{\epsilon} = \frac{K\bar{\epsilon}_f^{n+1}}{n+1} = \frac{K{\epsilon_h}^{n+1}}{n+1} \tag{2}$$

Note that if there is no hardening:

$$(n = 0 \text{ and } \bar{\sigma} = Y),\; w_i = Y\bar{\epsilon}_f = Y\epsilon_h$$

The external work (actual work) applied W_a is:

$$W_\alpha = F_e \Delta l \tag{3}$$

or per unit volume $(A_0 \times \Delta l)$ ***W***

$$w_\alpha = \frac{F_e \Delta l}{A_o \Delta l} = P_e \tag{4}$$

where P_e is the applied extrusion pressure.

For an ideal process, $w_a = w_i$, i.e.

$$P_e = \underbrace{\int_0^{\bar{\epsilon}_f} \bar{\sigma} d\bar{\epsilon}}_{\text{lower bound } P_e} = \frac{K\bar{\epsilon}_f^{n+1}}{n+1} = \frac{K{\epsilon_h}^{n+1}}{n+1} \tag{5}$$

In reality:

$$P_e \geq \int_0^{\bar{\epsilon}_f} \bar{\sigma} d\bar{\epsilon} = \frac{K\bar{\epsilon}_f^{n+1}}{n+1} = \frac{K{\epsilon_h}^{n+1}}{n+1} \tag{6}$$

Similar results can be obtained for rod or wire drawing (Fig. 2). The external work/volume in drawing is $w_a = F_d / A_f = s_d$ and so in general we have:

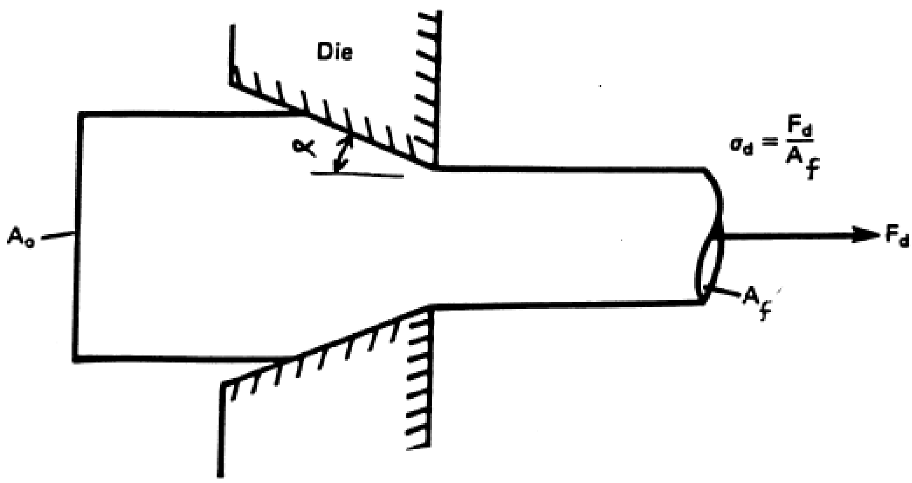

Figure 2: Illustration of rod or wire drawing.

$$\sigma_d \geq \int_o^{\bar{\epsilon}_f} \bar{\sigma} d\bar{\epsilon} = \frac{K\bar{\epsilon}_f^{n+1}}{n+1} = \frac{K{\epsilon_h}^{n+1}}{n+1} \tag{7}$$

where σ_d is the applied drawing stress.

Friction, Redundant Work and Efficiency

The actual work: $W_a = W_i + W_f + W_r$

Where W_f = Friction and W_r = Redundant (non-homogeneous deformation)
W_f and W_r are usually combined. We define the mechanical efficiency η as follows:

$$\eta = \frac{w_i}{w_a} \tag{8}$$

The efficiency η is a function of the die, lubrication, reduction rate, etc. Usually $0.5 < \eta < 0.65$.

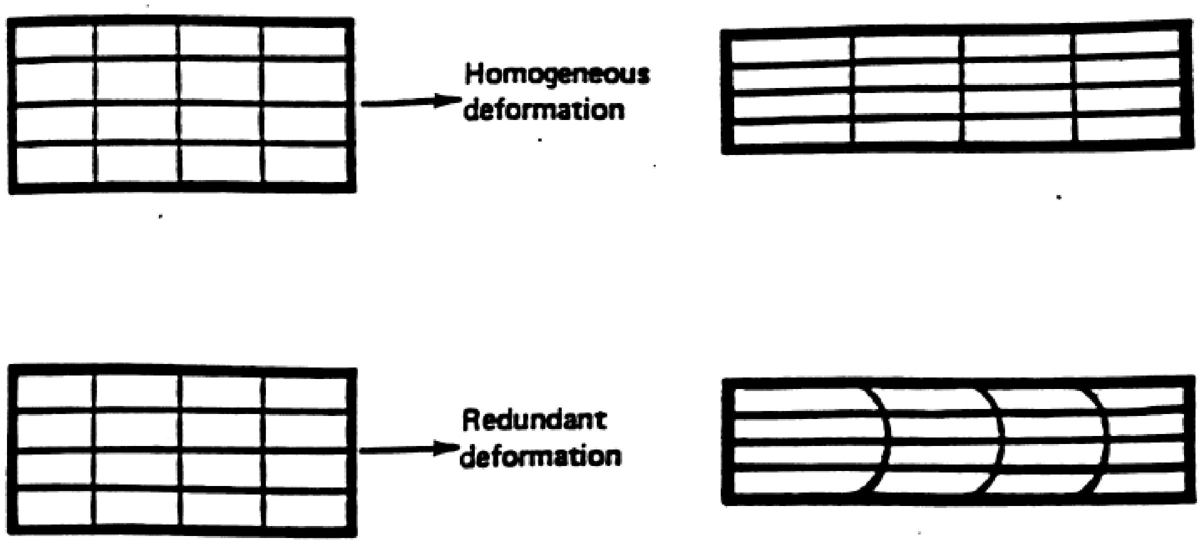

Figure 3: Comparison of ideal and actual deformation to illustrate the meaning of redundant deformation.

Generalizing the formulas given above for the extrusion pressure and drawing stress, we can write the following:

$$P_e = \frac{\int_0^{\bar{\epsilon}_f} \bar{\sigma} d\bar{\epsilon}}{\eta} = \frac{K\bar{\epsilon}_f^{n+1}}{(n+1)\eta} = \frac{K\epsilon_h^{n+1}}{(n+1)\eta} \tag{9}$$

And

$$\sigma_d = \frac{\int_0^{\bar{\epsilon}_f} \bar{\sigma} d\bar{\epsilon}}{\eta} = \frac{K\bar{\epsilon}_f^{n+1}}{(n+1)\eta} = \frac{K\epsilon_h^{n+1}}{(n+1)\eta} \tag{10}$$

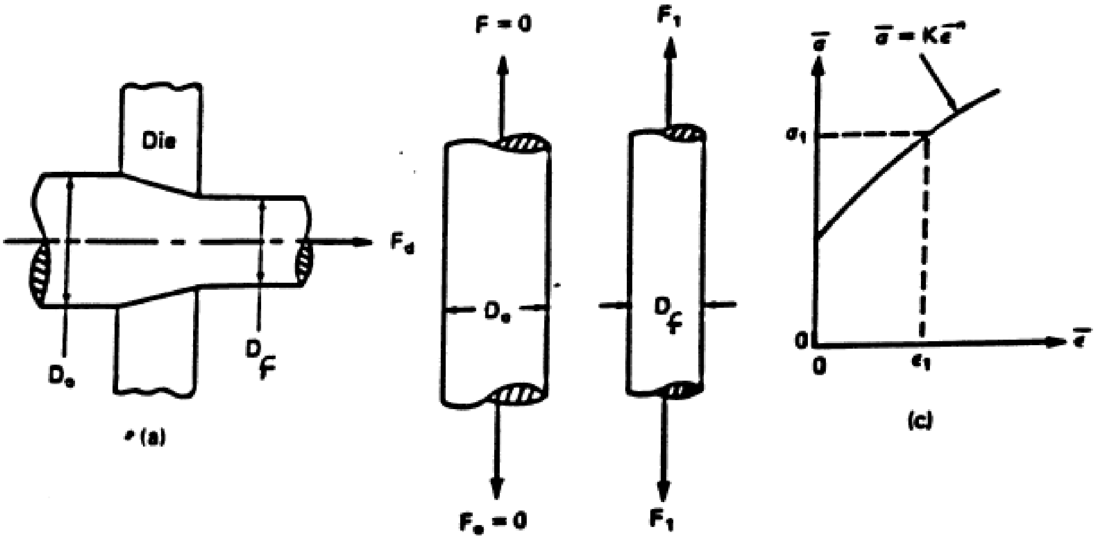

Figure 4: The stress-strain behavior is depicted in (c), the metal obeying $\bar{s} = K\bar{e}^{n}$, and s_1\ is to be considered as the true stress needed to reduce D_0 to D_f (e_1 is the corresponding true strain).

Example: A round rod of initial diameter, D_0 can be reduced to diameter $D\ f$ by pulling through a conical die with a necessary load, F_d, as shown in sketch 4(a). A similar result can occur by applying a uniaxial tensile load, as shown in sketch 4(b). Using the ideal-work method for both the drawing and tensile operations, compare the load F_d with the load F_1 (or the "drawing stress" s_d with the tensile stress s_1) needed to produce equivalent reductions. For drawing we showed that:

$$\sigma_d = \frac{K{\epsilon_h}^{n+1}}{n+1} \tag{11}$$

For tension,

$$\sigma_t = K{\epsilon_h}^{n} \tag{12}$$

From the two equations above,

$$\sigma_d/\sigma_t = \frac{\epsilon_h}{n+1} \tag{13}$$

But $e_h \le n$ (strain at ultimate load - max strain to avoid necking). So finally,

$$\sigma_d/\sigma_t = \frac{\epsilon_h}{n+1} \le \frac{n}{n+1} < 1 \tag{14}$$

Also,

$$\left. F_d = \sigma_d \frac{n}{4} D_f^2, F_t = \sigma_t \frac{n}{4} D_f^2 \right\} \rightarrow \frac{F_d}{F_t} < 1 \tag{15}$$

Maximum drawing reduction in axisymmetric drawing

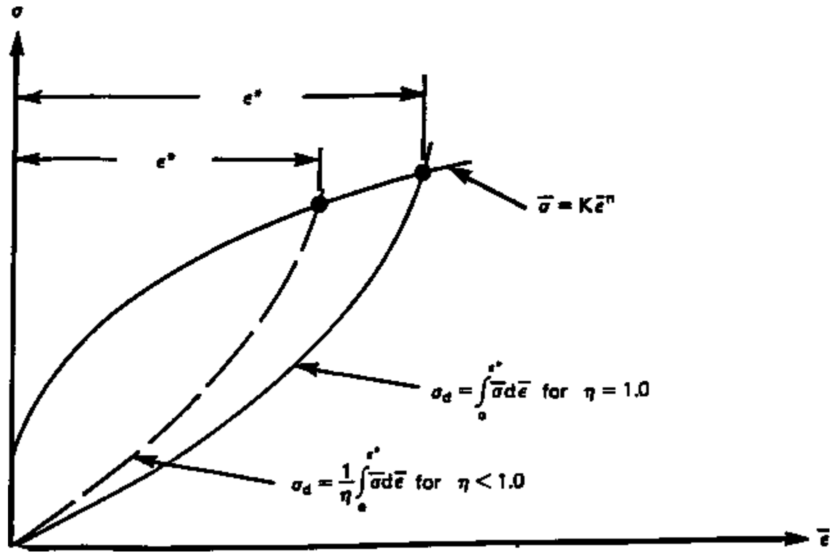

Figure 5: The tensile stress-strain curve and the drawing stress-strain behavior for two levels of deformation efficiency. The intersection points,e^*, are the limit strains in drawing.

With greater reduction the drawing stress *ad* increases. Its value can't be higher than the yield stress of the material at the exit.[1] ([1]The yield condition for axisymmetric problems has the form: $s_x + p$ =Y.S., where Y.S. is the yield stress of the material at any location inside the deformation zone and p the die pressure. Note that $p \geq 0$, which together with the yield condition at the exit implies that $s_d = s_x$ (at the exit) < Y.S. at the exit.) From the previous analysis:

$$\sigma_d = \frac{K\epsilon_h^{n+1}}{(n+1)\,\eta} \tag{16}$$

The maximum possible value of s_d is $K\bar{e}^n{}_{f*}$, where we denote as,

$$\epsilon_{f*} = \epsilon_{h*} = ln\frac{1}{1-r_{max}}$$

The final axial strain corresponding to maximum reduction. From the above equations:

$$K\epsilon_{h*}^n = \frac{K{\epsilon_{h*}}^{n+1}}{(n+1)\,\eta} \Longrightarrow \epsilon_{h*} = \eta(n+1) \tag{17}$$

With

$$\epsilon_{h*} = ln\frac{A_o}{A_{f*}} \Rightarrow \frac{A_o}{A_{f*}} = e^{\eta(n+1)}$$

and maximum reduction per pass

$$r_{max} = 1 - \frac{A_{f*}}{A_o} = 1 - e^{-\eta(n+1)} \tag{18}$$

For $h=1$ (perfect drawing) the maximum reduction is given as $r_{max}=1-e^{-n-1}$ and for n =0 (perfectly plastic material - no hardening) we have that: $r_{max}=1-e^{-1}$ = 63%.

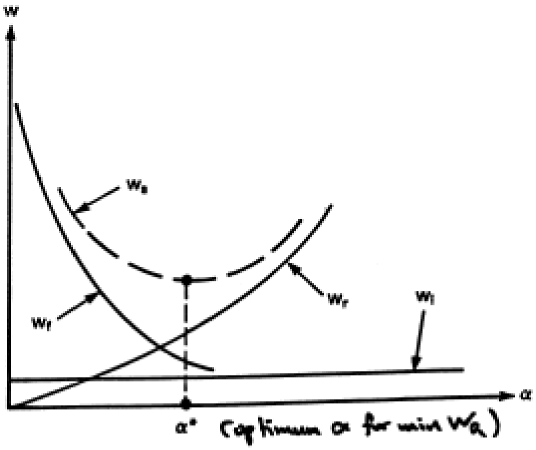

Figure 6: Influence of semi-die angle on the actual work W_a , during drawing where the individual contributions of ideal, W_i frictional, W_f, and redundant work, W_r , are shown.

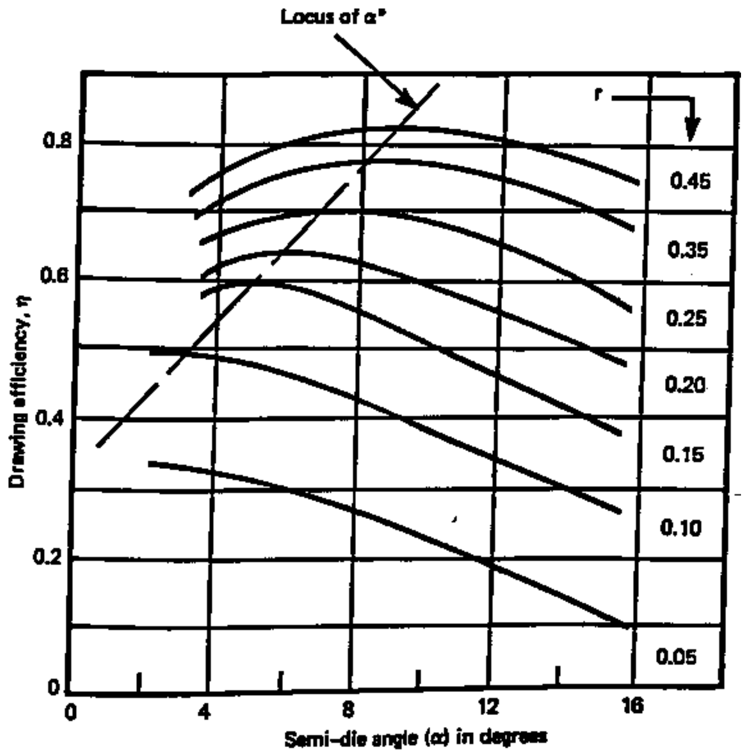

Figure 7: Effect of semi-die angle on drawing efficiency for various reductions; note the change in the optimal die angle a^*.

PLANE STRAIN EXTRUSION AND DRAWING

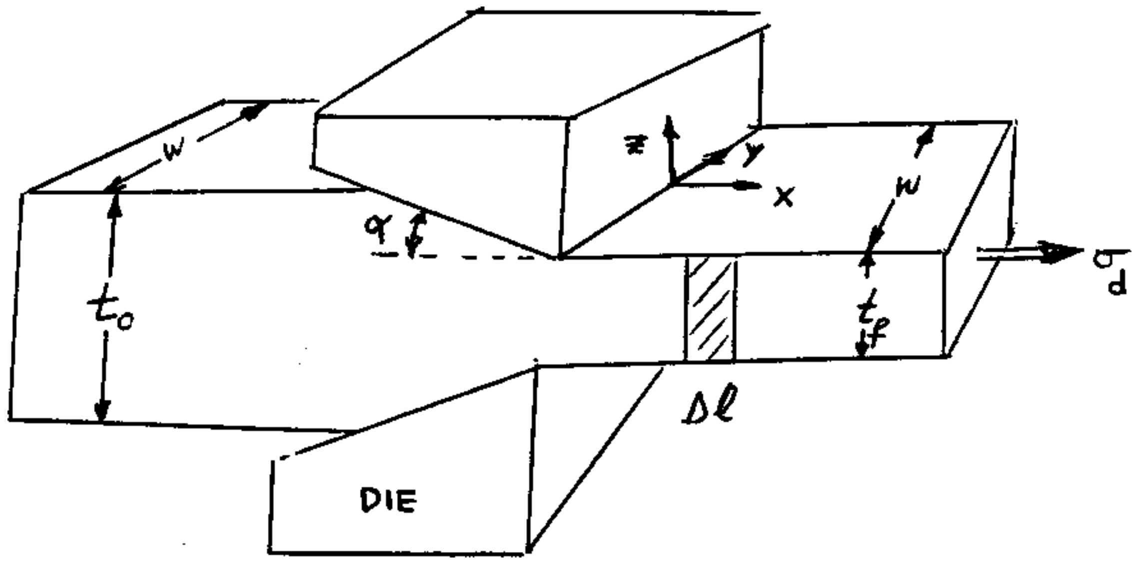

Figure 8: Plane strain drawing.

The calculations and previous definitions are applicable to plane strain problems with only minor modifications. The differences arise from the new form of the yield condition and the new expression for the equivalent strain. They have as follows:Yield condition:

$s_x + p = 2/\sqrt{3}\, Y.S.$, where Y.S. is the yield stress of the material at any location in the deformation zone.

Equivalent strain: $\bar{e} = 2/\sqrt{3}\, e_x$

The above changes will modify the final results as follows:

PLANE STRAIN EXTRUSION

Extrusion Pressure: $p_e = F/wt_o$

$$p_e = w_a = \frac{w_i}{\eta} = \frac{1}{\eta}\int_0^{\bar{\epsilon}_f} \bar{\sigma}\, d\bar{\epsilon} \qquad (19)$$

where $\bar{\epsilon}_f = \frac{2}{\sqrt{3}}\epsilon_h$, with the homogeneous strain $\epsilon_h = ln\frac{1}{1-r}$, where $r = \frac{t_o - t_f}{t_o}$.

For $\bar{\sigma} = Y$ (rigid plastic material): $p_e = \frac{Y\bar{\epsilon}_f}{\eta} = \frac{Y\frac{2}{\sqrt{3}}\epsilon_h}{\eta}$.

For $\bar{\sigma} = K\bar{\epsilon}^n$ (power law hardening): $p_e = \frac{K\bar{\epsilon}_f^{n+1}}{\eta(n+1)} = \frac{K(\frac{2}{\sqrt{3}}\epsilon_h)^{n+1}}{\eta(n+1)}$.

PLAIN STRAIN DRAWING:

Drawing Stress: $\sigma_d = \frac{F}{wt_f}$

$$\sigma_d = w_a = \frac{w_i}{\eta} = \frac{1}{\eta} \int_0^{\bar{\epsilon}_f} \bar{\sigma} d\bar{\epsilon} \tag{20}$$

where $\bar{\epsilon}_f = \frac{2}{\sqrt{3}} \epsilon_h$, with the homogeneous strain (x-strain) $\epsilon_h = ln \frac{1}{1-r}$, where $r = \frac{t_o - t_f}{t_o}$.

For $\bar{\sigma} = Y$ (rigid plastic material): $\sigma_d = \frac{Y \bar{\epsilon}_f}{\eta} = \frac{Y \frac{2}{\sqrt{3}} \epsilon_h}{\eta}$.

For $\bar{\sigma} = K \bar{\epsilon}^n$ (power law hardening): $\sigma_d = \frac{K \bar{\epsilon}^{n+1}}{\eta(n+1)} = \frac{K(\frac{2}{\sqrt{3}} \epsilon_h)^{n+1}}{\eta(n+1)}$.

For max reduction:

$$\sigma_d = \frac{2}{\sqrt{3}} (yield\ stress\ at\ exit) = \frac{2}{\sqrt{3}} K (\frac{2}{\sqrt{3}} \epsilon_h)^n \tag{21}$$

from which we finally conclude that:

$$r_{\max} = 1 - \exp \left[-\eta \left(n + 1 \right) \right] \tag{22}$$

Note that the max reduction is the same for both plane strain and axially symmetric problems.