Spring 2008 Lecture 5
WORK HARDENG

<u>Uniaxial Tension</u>:

Let us consider a uniaxial tensile test. As you will see in Module I of the laboratory, one collects load F versus deflection data l - l_0 during such a test on an Instron machine (Fig. 1, lecture 1). Here l_0 denotes the initial length of the specimen and l the instantaneous length when the applied load is F. The plot of F versus Δl = l - l_0 is shown in Fig. 1. Using the definition of the engineering stress S and engineering strain e = ${}^{\Delta l}/_{l_0}$, one can easily construct the S versus e diagram that looks identical in nature to the F versus Δl diagram (we effectively only need to change the scale of the F - Δl diagram to obtain the S - e diagram). For convenience, the plot S-e is shown on the same Figure 1.
In Figure 1 you should notice all critical regions and points: (i)the elastic region and the (initial)yield point (yield stress Y)defining the transition from the region of elastic (re-coverable)to elasto-plastic (non-recoverable)deformations, (ii)the region of uniform plastic deformation, (iii)the region of non-uniform plastic deformation and (iv)the point where necking is initialized. Note that in the region of uniform plastic deformation, the strengthening effect offsets the area reduction. After the onset of necking, the deformation becomes localized in the necking region.

We define the ductility as:

$$\% \text{ elongation at fracture} : \frac{l_f - l_0}{l_0} \times 100 \qquad (1)$$

Or using area as

$$\% \text{ area reduction at fracture} : \frac{A_0 - A_f}{A_0} \times 100 \qquad (2)$$

Usually as the strength increases, ductility decreases.
We define the <u>ultimate stress</u> S_u as the engineering stress at the onset of necking (i.e. at the point of maximum load in a tensile test).

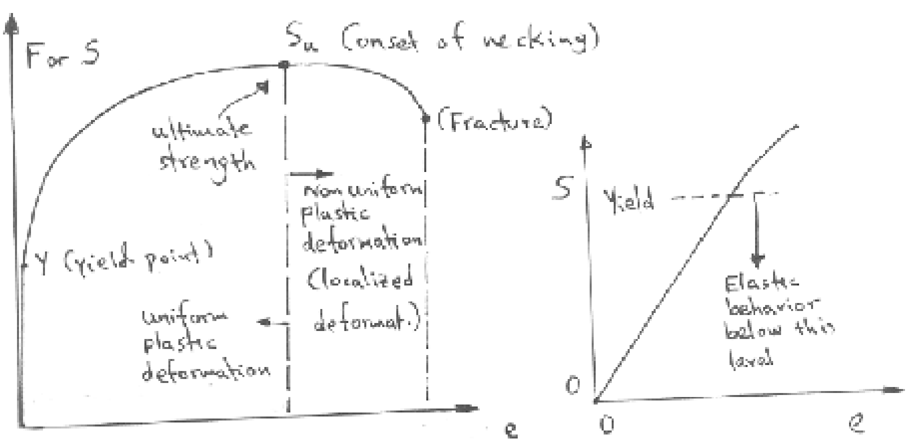

Figure 1: (a)The load versus deflection or engineering stress versus engineering strain diagram obtained in a uniaxial tensile experiment for a ductile metal (b)To clearly show the transition from elastic (recoverable) to elastoplastic (non-recoverable)deformations, the diagram on the left

is reproduced for small strains e. When plotting stress versus strain using strain scales for large deformations, the elastic region is so small that the stress/strain curve in this region looks almost vertical!

The 0.2% yield strength

Figure 2 defines the 0.2% yield strength using the offset method.

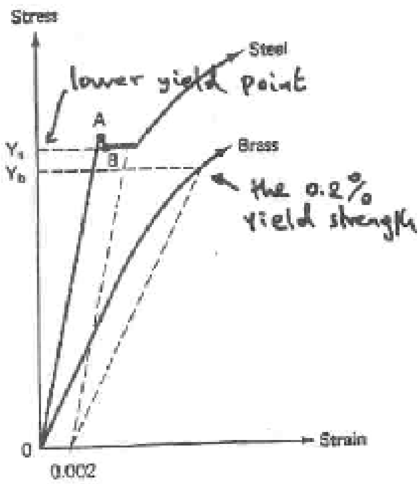

Figure 2: The 0.2% yield strength is the stress at which a 0.2% permanent offset occurs. This definition simplifies the clarification of yield point for cases without a distinct transition from elastic to plastic regions (e.g. lower and upper yield points for certain steels, etc.).

Figure 3 shows again the engineering stress-engineering strain curve in uniaxial tension and the state of the specime at the various straining levels.

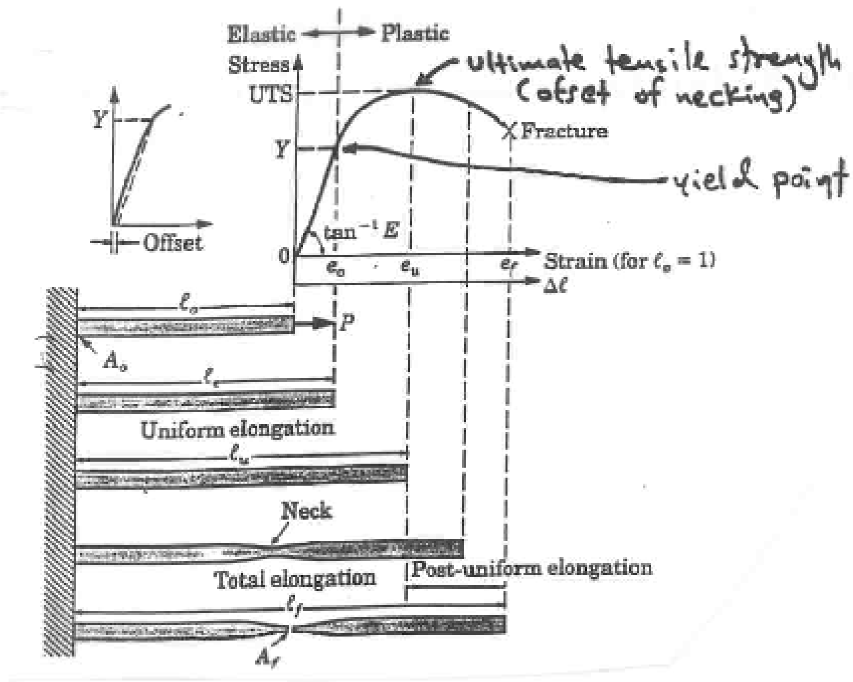

Figure 3: Engineering stress/strain diagram showing the region of uniform deformation, the initiation of necking and the post-uniform deformation up to the point of fracture.

The true stress σ - true strain e curve

Figure 4 shows the engineering stress/engineering strain diagram designed directly from uniaxial load/deflection data. As we discussed earlier, engineering strain and engineering stress are not very appropriate for the regime of large deformations. To transform the $S - e$ diagram to an $s - e$ diagram, we need to make use of the following identities:

$$\sigma = \frac{F}{A} = \frac{F}{A_0}\frac{A_0}{A} = S\frac{l}{l_0} = S(1+e), \quad \text{(using } lA = l_0A_0 \text{ (incompressibility))} \tag{3}$$

and

$$\epsilon = ln\frac{l}{l_0} = ln(1 + \frac{l - l_0}{l_0}) = ln(1+e) \tag{4}$$

Both of the above equations are valid only up to the point of necking. Using these equations, the $S - e$ diagram of Fig. 4 can be transformed to the $s - e$ plot of Fig. 5. Note the big differences in between Figures 4 and 5. The $s - e$ diagram does not have a maximum (as the S-e plot has)and the stress σ increases monotonically.

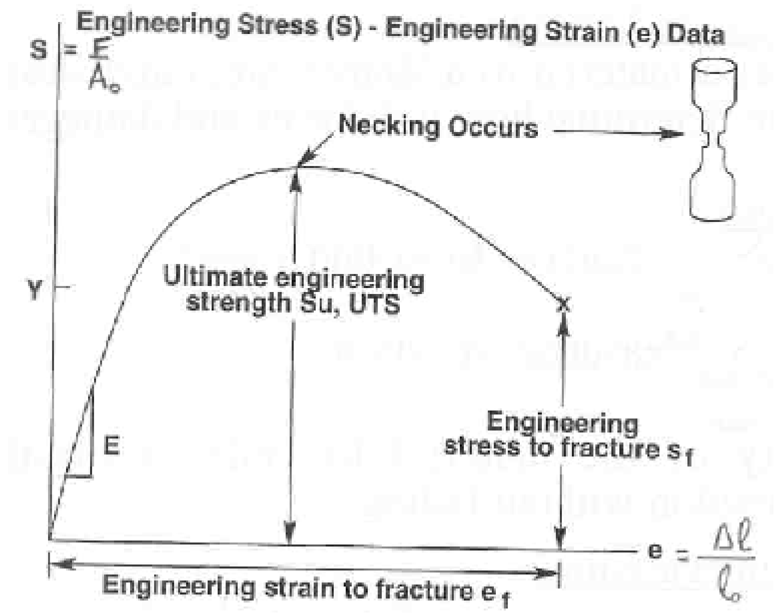

Figure 4: Engineering stress S versus engineering strain e plot obtained directly from uniaxial load F-deflection Δl data.

Using the incompressibility condition lxA = constant, one can write that:

$$ldA + dlA = 0, \quad \text{or} \quad \frac{dl}{l} = -\frac{dA}{A} \tag{5}$$

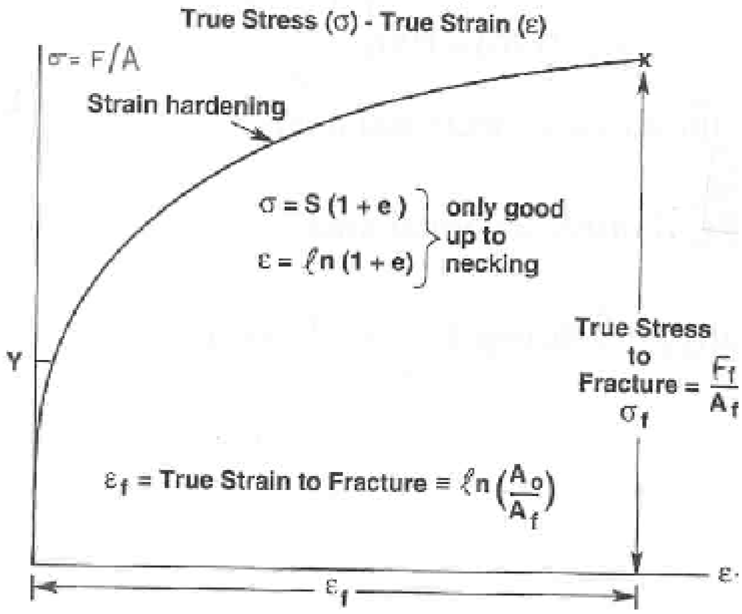

Figure 5: The true stress-true stain diagram obrained using the plot of Fig. 4 and equations (3)and (4).

Recall the definition of the true strain $e = \ln l / l_0$ Differentiating this equation and together with equation (5), we arrive at the following very useful expression for the increment de of the true strain:

$$d\epsilon = \frac{dl}{l} = -\frac{dA}{A} \tag{6}$$

It is customary to define the level of deformation using the percentage area reduction *r*:

$$r = \frac{A_0 - A}{A_0} \times 100\% \tag{7}$$

Another useful relation is introduced here providing a relation between the true strain e and the %-area reduction r = (A_0- A)/A_0, one can show that:

$$\epsilon = ln\frac{A_0}{A} = ln\frac{A_0}{(1-r)A_0} = ln\frac{1}{1-r} \tag{8}$$

Based on the above very useful equation, one can also derive the area reduction in terms of true strain**:**

$$r = 1 - e^{-\epsilon} \tag{9}$$

Loading and Unloading in a Tensile Test:

Let us consider the uniaxial stress/strain curve of Fig. 6. If we start from no load (point A) and we load the specimen up to stress s_1 (point B), the new yield stress becomes s_1. To understand this, remove the load after you reach point B. The unloading process (line BC)is elastic. After you reload specimen C, the material behaves elastically until you reach point B. The original $s - e$ curve is followed after point B.

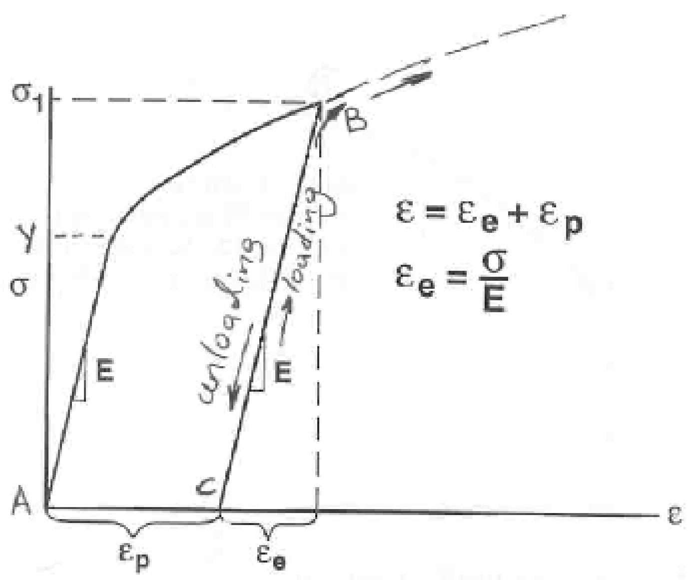

Figure 6: The *a* versus e curve shown here emphasizes that the 'yield stress' increases as you deform the material, e.g. *Y* as shown here is the yield stress of an initially underformed material, where as s_1 is the 'yield stress' of the material that you loaded in tension up to point B (strain e). Loading and unloading in a tensile test is also used here to define the elastic e_e (recoverable) and plastic (permanent) part e_p of the total strain e .

The $e = e_e + e_p$ Decomposition: Neglecting Elastic Deformation

Figure 7 is used to define via an unloading process the decomposition of the strain in elastic and plastic parts:

$$\epsilon = \epsilon_e + \epsilon_p \tag{10}$$

When interested in large strains (e.g. in metal forming processes), we can neglect the elastic strain e_e and approximate:

$$\epsilon \approx \epsilon_p \tag{11}$$

We will use the above approximation most of the time in the remaining of this course!

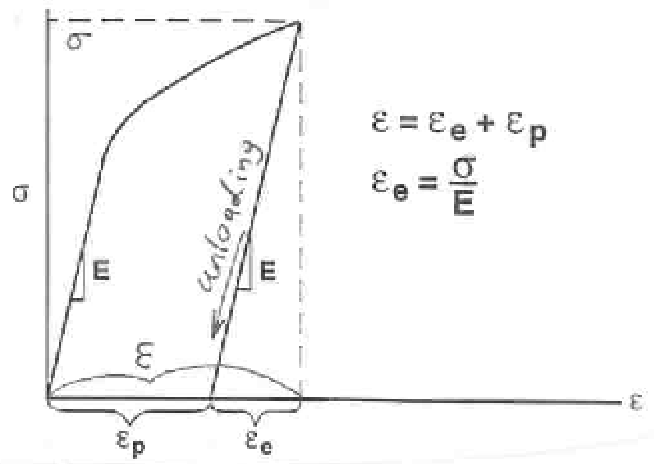

Figure 7: The $e = e_e + e_p$ decomposition. Hooke's law is still a valid law but you should notice that it only relates stresses s and <u>elastic</u> strains e_e. We will need a new set of equations to define the relation of the plastic strains e_p with the stresses s .

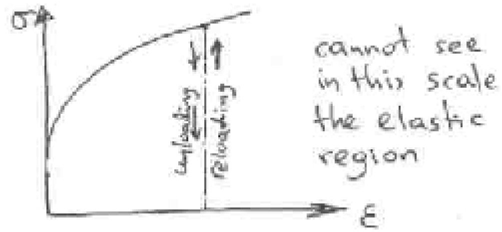

Figure 8: In the large deformation regime, we usually neglect elastic deformations (they are too small) and thus the unloading curve looks almost a vertical line in the s —large e plot. Also, in such a diagram the initial elastic region is not visible and the stress curve jumps vertically to the initial yield stress *Y.*

An example of a hardening law: Power law strain hardening: $\sigma = Ke^n$.

To simplify the representation of the hardening behavior, it is customary to curve-fit the $s - e$ data. The simplest expression that will be used in this course is of the form:

$$\sigma = K\epsilon^n, \quad \text{where } n \;=\; \text{strain hardening exponent} \;\;\&\;\; K = \text{strength coefficient} \qquad (12)$$

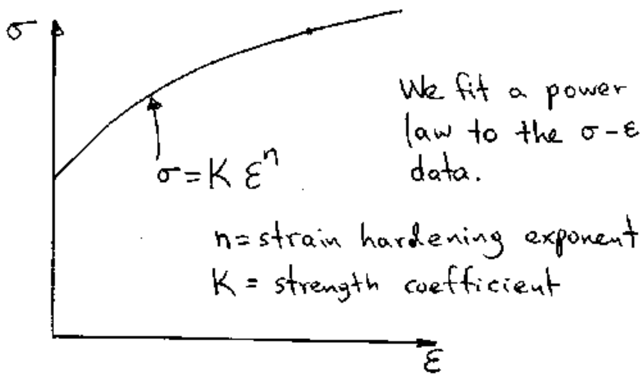

Figure 9: The power law hardening approximation. The strain e shown here is the total strain and we assume that $e = e_p$.

Note that the model of equation (12) results in a 'line' in a *logσ-loge* plane (see Fig. 10):

$$log\sigma = logK + nlog\epsilon \qquad (13)$$

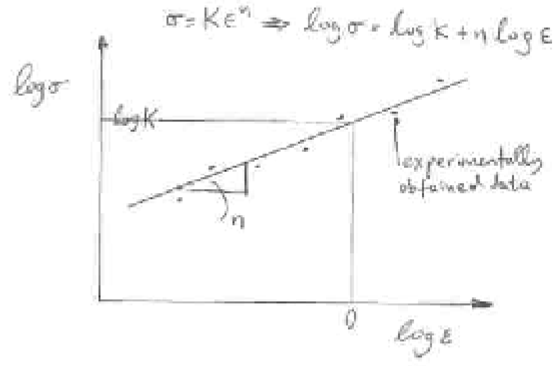

Figure 10: Determination of K and n for a power law hardening model by plotting $log\sigma$ versus $log\epsilon$.

Other Examples of Hardening Laws: Rigid-Plastic (no hardening), Linear Hardening.

Figure 11 shows two other typical work hardening material models of interest to this class; the case of no-hardening (rigid-plastic material model) and the case of linear hardening. The initial yield stress Y for these models (including the power-law model) is shown as well.

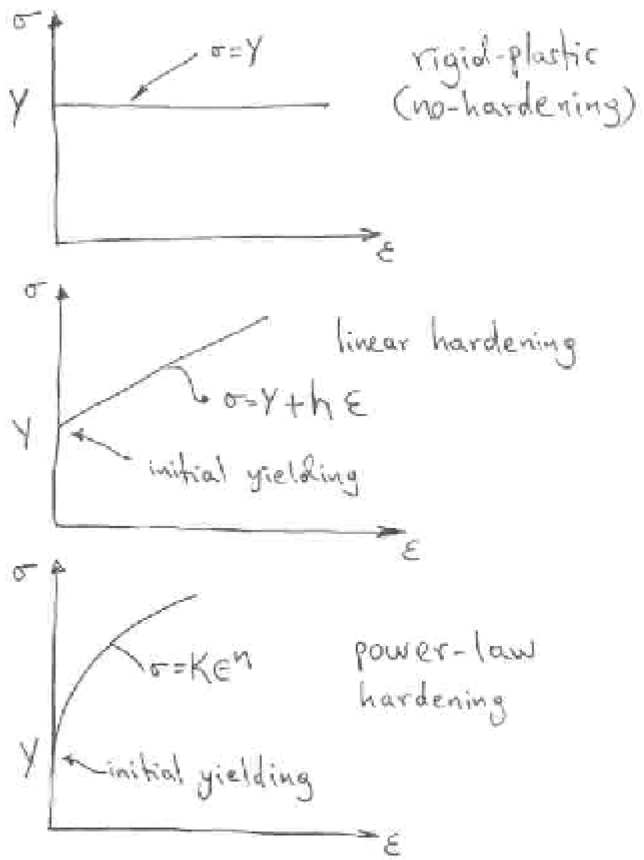

Figure 11: (a)Rigid-plastic (no hardening), (b)linear hardening and (c)power-law hardening models.

Tensile Instability

Let us consider a general hardening behavior $\sigma(e)$ (i.e. σ being some function of the strain e). At the onset of necking, the force is maximum:

$$\text{At the ultimate point:} \quad dF = 0 \tag{14}$$

Using $F = \sigma A$, the above equation is simplied as follows:

At the ultimate point : $d(\sigma A)=0$, or $\sigma dA + d\sigma A = 0$, or

$$\frac{d\sigma}{\sigma} = -\frac{dA}{A} \tag{15}$$

Using equation (6), we can simplify the above equation as follows:

$$\text{At the ultimate point :} \quad \frac{d\sigma}{\sigma} = d\epsilon, \quad \text{or} \quad \frac{d\sigma}{d\epsilon} = \sigma \tag{16}$$

The above equation is valid at the ultimate point for any true stress/true strain relation.

True strain e_u at the ultimate point for a power law model: $\sigma = Ke^n$

For the particular case of $\sigma = Ke^n$, we compute $\sigma = Kne^{n-1}$ and equation (16) is simplified as follows:

$$\text{At the ultimate point :} \quad Kn\epsilon_u^{n-1} = K\epsilon_u^n \tag{17}$$

The true strain at the ultimate point is thus given as follows:

True strain at the ultimate point for a power law material : $e_u = n$
(18)

The ultimate stress S_u for a power law model: $S_u = K(\underline{n/e})^n$

Let F_u be the maximum load (at the ultimate point): $S_u = F_u/A_0$. Recall that the ultimate stress is an engineering stress (force per unit initial area). Using the result of equation (18), we can compute the true stress σ_u at the ultimate point as follows:

$$\sigma_u = K\epsilon_u^n = Kn^n \tag{19}$$

Using the definition of true stress $e = F / A$

$$F_u = \sigma_u A_u = Kn^n A_u \tag{20}$$

But from the definition of true strain: $e = ln\, A/A_0$ we conclude that:

$$A_u = A_0 e^{-\epsilon_u} = A_0 e^{-n} \tag{21}$$

Combining equations (20)and (21), we finally derive:

$$S_u = \frac{F_u}{A_0} = \frac{Kn^n A_0 e^{-n}}{A_0} = K(\frac{n}{e})^n, \quad (e = \text{ base of natural logarithms!}) \tag{22}$$

How do you account for the effects of cold-working (hardening) that may have been induced initially (by prior processing)?

Let us consider a metallic specimen that contains no effects of work hardening prior to the tensile deformation that is currently undergoing through. We use as an example $a = Ke^n$ to describe the hardening behavior characteristics in this tension test (here called test I, Fig. 12).

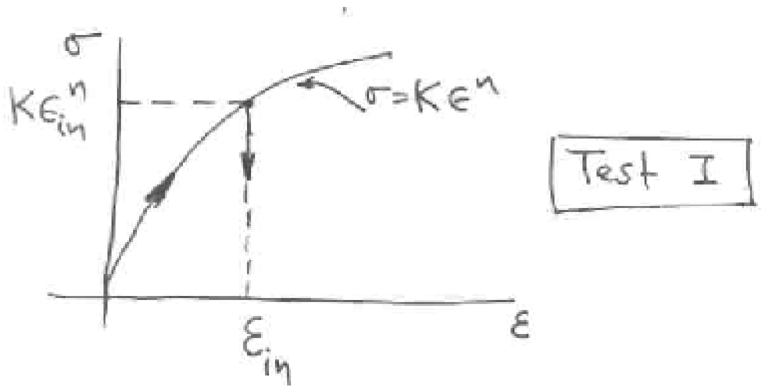

Figure 12: Test I: Uniaxial straining of the workpiece up to strain e_{in}. At the end of this test after unloading we obtain a specimen with e_{in} permanent deformation.

Let us load this specimen up to a strain e_{in} (see Fig. 12). We now remove the load (i.e. we unload). There is an e_{in} permanent deformation left over in the specimen at the end of this test.

Let us now take the specimen resulted from test I and load it again in tension: What is the stress/strain relation resulting from this second test? As it is clear from Fig. 13, the answer is:

$$\sigma = K(\epsilon_{\text{in}} + \epsilon)^n \tag{23}$$

The key idea here is that the hardening law (e.g. $a = Ke^n$) is a material property independent of a particular test and the resulting yield (or flow)stress a needs to be always computed using the <u>total</u> (plastic)strain imposed on the material.

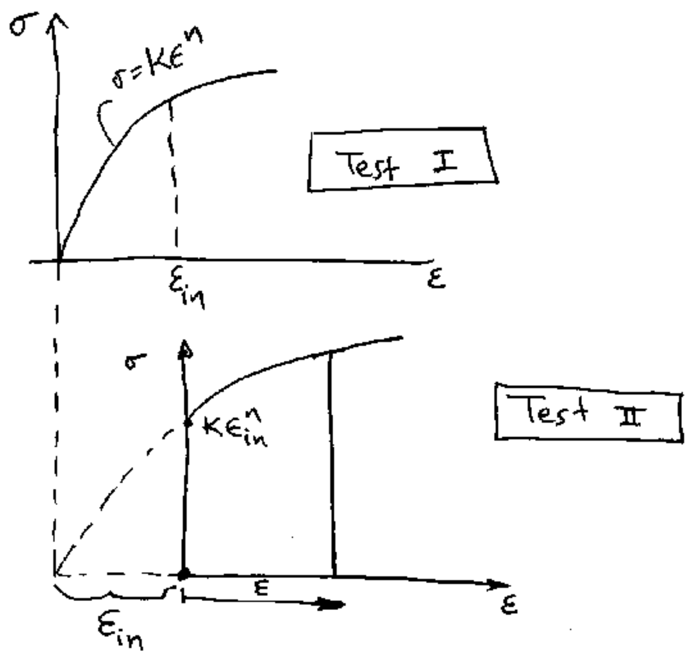

Figure 13: Test II: Uniaxial straining of the workpiece that was earlier (test I)deformed up to strain e_{in}. Note that the material obeys the power law in both tests I and II, but the strain used in this hardening law is the <u>total</u> strain induced in the material.